Practice questions for 411-3 final: solutions

Problem 1. First consider the RA model. Given that the real interest rate after date 0 is kept the same (and that we select the equilibrium with the same consumption at infinity), the Euler equation implies $dY_1 = dC_1 = 0$. Then we can write the Euler equation

$$C_0^{-\sigma} = \beta(1+r_0)C_1^{-\sigma}$$

and log-linearize to obtain the consumption effect (and output effect, since this model has consumption as the only part of demand):

$$dY_0 = dC_0 = -C\sigma^{-1}\frac{dr_0}{1+r} \tag{1}$$

How much of this is the "direct" effect of rates vs. the "indirect" effect of income or taxes? There is no tax effect here, since steady-state debt is zero. For the income effect, consider that the representative-agent household consumes a fraction $1-\beta$ of the present value of its income; therefore, the "indirect" effect of income is precisely $1-\beta$ of dY_0 in (1), and the "direct" effect is the remaining fraction β.

Now consider the TA model. We know that it has exactly the same output effect as in (1): as discussed in class, if households hold zero assets in equilibrium anyway, replacing some of them with "hand-to-mouth" households required to hold zero assets does not change the equilibrium. The breakdown between indirect and direct effects, however, changes. The "indirect" effect of income is now the $1-\beta$ of dY_0 (as in the RA model) consumed by the fraction $1-\mu$ of non-hand-to-mouth households, plus the entirety of dY_0 consumed by the fraction μ of hand-to-mouth households, or

$$((1-\mu)(1-\beta)+\mu)\, dY_0$$

and the "direct" effect of rates is dY_0 minus this, or

$$(1-\mu)\beta dY_0$$

which is attenuated relative to the direct effect in the RA case by the fraction $1-\mu$; this reflects the fact that only the non-hand-to-mouth households respond directly to the change in rates in the TA model.

If the steady-state level of government debt is not zero, then there is also an income effect of the change in rates, and a tax effect. The interest rate change affects date-1 income by Bdr_0, where B is steady-state bonds. In present-value terms at date 0, this effect is $B\frac{dr_0}{1+r}$, and then in the RA model a fraction $(1-\beta)B\frac{dr_0}{1+r}$ of this is spent at date 0. But then the government will also have to raise the same present value of revenue in additional taxes, which will have an offsetting effect $-(1-\beta)B\frac{dr_0}{1+r}$ on spending. The Euler equation still implies the same output effect (1), just now with a direct effect with an added $(1-\beta)B\frac{dr_0}{1+r}$ due to interest income and an offsetting indirect effect from taxes of $-(1-\beta)B\frac{dr_0}{1+r}$.

In the TA model, assuming that taxes are levied on both the permanent-income and hand-to-mouth agents, then the tax effect will be nonzero. As we saw in class (the "two-agent model" slide of lecture 8, starting on slide 15 of the pdf), the response of the TA model to monetary policy in the general case is the same as in the RA model, except with the added term $-\frac{\mu}{1-\mu}d\mathbf{T}$ to reflect changes in taxes—which, here, as taxes are cut due to the decline in interest expense, will be positive.[1] This term reflects a change in the direct effect of taxes (now concentrated in the period of the tax cut, as the hand-to-mouth households consume

[1] We can't say more since I didn't specify what the fiscal rule was—e.g. whether taxes are cut immediately or later.

more) and also an indirect effect of output (which is responsible for the $1-\mu$ in the denominator, which acts as a multiplier).

Problem 2. If we write

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} - \mathbf{M} d\mathbf{T} + \mathbf{M} d\mathbf{Y}$$

then in response to a real interest rate cut, all three terms are generally positive at date 0: the "direct effect" $\mathbf{M}^r d\mathbf{r}$, the indirect effect from taxes $-\mathbf{M}d\mathbf{T}$, and the indirect effect from output $\mathbf{M}d\mathbf{Y}$.

In present value, $\mathbf{M}^r d\mathbf{r}$ will be negative, because the present value of change in consumption must in response to a shock must equal the present value of changes in income (including interest income on bonds), and the decline in rates means a decline in income on bonds. There is an offsetting positive present value effect of $-\mathbf{M}d\mathbf{T}$. Also, $\mathbf{M}d\mathbf{Y}$ will also generally have positive present value, because dY_t will be positive at all or nearly all periods.

If we further split up $\mathbf{M}^r d\mathbf{r}$ into substitution and income terms as in class, so that we have

$$d\mathbf{Y} = \mathbf{M}^{r,sub} d\mathbf{r} + \mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M} d\mathbf{T} + \mathbf{M} d\mathbf{Y}$$

then $\mathbf{M}^{r,sub} d\mathbf{r}$ will be positive at date 0 but have zero net present value (since it perturbs the Euler equation without perturbing the budget constraint, so that the present value of consumption can't change). Meanwhile, $\mathbf{M}^{r,inc} d\mathbf{r}$ will be negative both at date 0 and in present value, since the direct income effect of a cut in real interest rates is purely negative.

If we look at $\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T}$ together, then there will be zero present value effect, since the decrease in income from higher real interest rates exactly equals the decrease in taxes. The date-0 effect depends on whether there is a larger immediate consumption effect from cuts in interest income vs. cuts in taxes, which in turn depends on whether average MPCs are higher for the average interest earner vs. the average income earner (since change income proportionally in the basic model). Generally, average MPCs will be higher for the latter, since they decrease more rapidly in assets than in income, so that we expect the date-0 effect of $\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T}$ to be positive: the positive consumption effect of the cut in taxes outweighs the negative consumption effect of falling interest income.

Finally, if we split the GE effects as in class as

$$d\mathbf{Y} = \mathcal{M}\mathbf{M}^{r,sub} d\mathbf{r} + \mathcal{M}(\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M} d\mathbf{T})$$

then we generally expect both terms to be positive: since both $\mathbf{M}^{r,sub} d\mathbf{r}$ and $\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T}$ have an increase in date-0 spending (and an offsetting decrease in present value terms later), their general equilibrium effect will be a boom, just like a deficit-financed fiscal shock leads to a boom.

If taxes at the margin are assessed lump-sum in proportion to assets, then the taxes would exactly offset in the income effect of interest rates (the same people would be receiving interest income and paying taxes to finance it at the margin), so $\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T}$ would be zero and we'd be left just with $\mathcal{M}\mathbf{M}^{r,sub} d\mathbf{r}$.

Alternatively, if the tax cut is delayed for many periods, then the positive effect of $-\mathbf{M}d\mathbf{T}$ would be delayed for many periods, and it no longer would be clear that $\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T}$ is positive at date 0 nor that $\mathcal{M}(\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T})$ is positive in general.

Problem 3. The $\mathbf{E}$ matrix for this friction looks like

$$\mathbf{E} = \begin{pmatrix} 1 & 1-\theta & 1-\theta & 1-\theta & 1-\theta \\ 1 & 1 & 1-\theta & 1-\theta & 1-\theta \\ 1 & 1 & 1 & 1-\theta & 1-\theta \\ 1 & 1 & 1 & 1 & 1-\theta \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

i.e. that $E_{ts} = 1$ for $t \geq s$ and $E_{ts} = 1 - \theta$ for $t < s$, because prior to a shock being realized, only $1 - \theta$ have updated expectations about it.

We note that this $\mathbf{E}$ matrix equals $(1-\theta)\mathbf{E}^{RE} + \theta\mathbf{E}^{myo}$, where the "rational expectations" $\mathbf{E}^{RE}$ has all 1s, and the "myopic" $\mathbf{E}^{myo}$ has all 1s on and below the main diagonal and 0s above it.

Since the modified $\mathbf{M}$ matrix is linear in $\mathbf{E}$, we can obtain the modified $\mathbf{M}$ matrix from this friction by taking the respective combination of $(1-\theta)$ times the rational expectations $\mathbf{M}$ matrix (which we've been provided) and θ times the myopic $\mathbf{M}$ matrix—where the latter, as we saw in class, is just the first column repeatedly shifted down, so that for the $\mathbf{M}$ matrix here it is

$$\mathbf{M} = \begin{pmatrix} 3/4 & 0 & 0 & \ddots \\ 1/4 & 3/4 & 0 & \ddots \\ 0 & 1/4 & 3/4 & \ddots \\ 0 & 0 & 1/4 & \ddots \\ 0 & 0 & 0 & \ddots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

i.e. you react to a change in income at every date the same as you'd react to a surprise at date 0.

Taking a mixture of $1 - \theta = 1/2$ the original RE matrix and $\theta = 1/2$ this, we get a modified

$$\widetilde{\mathbf{M}} = \begin{pmatrix} 3/4 & 1/8 & 0 & \ddots \\ 1/4 & 5/8 & 1/8 & \ddots \\ 0 & 1/4 & 5/8 & \ddots \\ 0 & 0 & 1/4 & \ddots \\ 0 & 0 & 0 & \ddots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Intuitively, the reaction to a shock is now a mixture of $1/2$ of agents reacting with rational expectations and $1/2$ of agents reacting myopically.

One can also derive this answer by applying our formula from lecture 9, which states that

$$\widetilde{M}_{t,s} = \sum_{\tau=0}^{\min(t,s)} (E_{\tau,s} - E_{\tau-1,s}) M_{t-\tau,s-\tau}$$

In this context, we note that $E_{\tau,s} - E_{\tau-1,s}$ is only nonzero for $\tau = s$, where it equals θ, and $\tau = 0$, where it equals $1-\theta$ (in the case where $s=0$ then the two are summed). Therefore, we have $\widetilde{M}_{t,s} = \theta M_{t-s,0} + (1-\theta)M_{t,s}$ for $t \geq s$ and $\widetilde{M}_{t,s} = (1-\theta)M_{t,s}$ for $t < s$. This gives the same values for $\widetilde{\mathbf{M}}$ as above.

Problem 4. The Leontief inverse is $\Psi = (I-\Omega)^{-1}$, and thus if I replace Ω with $\widetilde{\Omega} \equiv \mu I + (1-\mu)\Omega$, the corresponding Leontief inverse is

$$\begin{aligned}\widetilde{\Psi} &= (I - \mu I - (1-\mu)\Omega)^{-1} \\ &= (1-\mu)^{-1}(I-\Omega)^{-1} \\ &= (1-\mu)^{-1}\Psi\end{aligned}$$

so that it's just scaled up by $(1-\mu)^{-1}$: the additional "roundaboutness" scales up the exposure of every sector to every other sector, including itself. Since the Domar weights are the first row of the Leontief matrix, they are also scaled up by λ_i.

In our economy, Hulten's theorem says that $d\log(C/L) = \sum_i \lambda_i d\log A_i$, so scaling up the Domar weights by $(1-\mu)^{-1}$ also scales up the effect of any given productivity shock on final output C (holding L fixed) by $(1-\mu)^{-1}$. Intuitively, because production in the economy involves more input-output relationships in the more "roundabout" economy, where a given productivity shock will translate output into more input, the impact of productivity shocks is higher.

Conceptual questions.

Problem 5. Sticky expectations can more easily generate hump-shaped impulse responses, since they delay agents' responses to the change in expectations starting from a shock at date 0. For instance, if there is a persistent AR(1) cut in real interest rates, but households have sticky expectations, the response at date 0 may be small, and then increase as more households update their expectations about rates, before falling again—a hump-shaped response.

Cognitive discounting does not have this same tendency, because it attenuates households' expectations of future deviations from steady state in every period, no matter how much time has elapsed since the shock hit at date 0.

Problem 6. If we increase the probability λ of "free resets" (which which lead to price changes with probability 1), then to hit the same overall frequency of price changes, we need to make price changes conditional on no free reset less likely. This implies raising the menu cost ξ, and thereby widening the steady-state adjustment bands $[-\bar{x}, \bar{x}]$.

In general, we saw in class (i.e. in our Jupyter notebook when comparing the model with free resets to the one without) that this implies a higher weight on the intensive margin and lower weight on the extensive margin in the equivalent TD mixture. Intuitively, this is because for free resets, there is only an intensive margin, and no extensive margin; so as free resets become a larger share of overall price changes, the weight on the intensive margin rises.

Problem 7. It has anti-inertia. More flexible sectors will raise prices more than stickier sectors leading up to a positive real marginal cost shock. After the shock has passed, the relative price gap needs to close. More flexible sectors will cut prices and stickier sectors will raise prices, but the more flexible sectors will do more of this (because they're more flexible), and thus we'll see a decline in overall prices, leading to anti-inertia in the price index.