THE “STANDARD INCOMPLETE MARKETS” MODEL

Econ 411-3
Matthew Rognlie, Spring 2024

INDIVIDUAL HOUSEHOLD PROBLEM

THE "STANDARD INCOMPLETE MARKETS" MODEL

- In sequential form, household i solves

$$\max_{\{a_{it}, c_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it})$$

subject to period-by-period budget constraint

$$a_{it} + c_{it} = (1+r)a_{i,t-1} + y(e_{it})$$

and some borrowing constraint

$$a_{it} \geq \underline{a}$$

- Exogenous state e follows Markov chain, initial assets $a_{i,-1}$ taken as given, standard assumptions on u (assume, e.g., CRRA), incomes $y(e)$ bounded away from zero

FROM SEQUENTIAL FORM TO BELLMAN EQUATION

$$\max_{\{a_{it},c_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it})$$

$$a_{it} + c_{it} = (1+r)a_{i,t-1} + y(e_{it})$$

$$a_{it} \geq \underline{a}$$

$$V(e,a) = \max_{c,a'} u(c) + \beta \mathbb{E}[V(e',a') \,|\, e]$$

$$s.t.\ a' + c = (1+r)a + y(e)$$

$$a' \geq \underline{a}$$

Two state variables: exogenous state e, endogenous assets a

SOLVING THE BELLMAN EQUATION: OPTIMAL POLICIES

$$V(e,a) = \max_{c,a'} u(c) + \beta \mathbb{E}[V(e',a') \,|\, e]$$

$$s.t.\ a' + c = (1+r)a + y(e)$$

$$a' \geq \underline{a}$$

*Solved by **policies***

$$a'(e,a) \quad c(e,a)$$

*Policy functions satisfy standard **first-order condition**:*

$$u'(c) \geq \beta \mathbb{E}[V_a(e',a') \,|\, e]$$

where equality holds unless borrowing constraint binds

*Can obtain derivative on right from **envelope condition**:*

$$V_a(e,a) = (1+r)u'(c)$$

TAKING STOCK OF SOLUTION

- Each period, we have first-order and envelope conditions:

$$u'(c) \geq \beta \mathbb{E}[V_a(e', a') \,|\, e] \qquad\qquad V_a(e, a) = (1 + r)u'(c)$$

- Combine (writing policy functions explicitly, a bit tedious) as

$$u'(c(e, a)) \geq \beta(1 + r)\mathbb{E}[u'(c(e', a'(e, a)) \,|\, e]$$

- Back in sequential notation, becomes standard Euler equation

$$u'(c_{it}) \geq \beta(1 + r)\mathbb{E}_t[u'(c_{i,t+1})]$$

FIRST RESULT: FOR FINITE ASSETS, $\beta(1+r) < 1$

- Euler equation for sequence form

$$u'(c_{it}) \geq \beta(1+r)\mathbb{E}_t[u'(c_{i,t+1})]$$

- If $\beta(1+r) \geq 1$, then $u'(c_{it})$ is "supermartingale", since then

$$\mathbb{E}_t[u'(c_{i,t+1})] \leq u'(c_{it})$$

  i.e. in expectation it's decreasing

- Supermartingale convergence theorem: if bounded, then $u'(c_{it})$ will converge almost surely to some random variable u'^*

RESULT CONTINUED

- Supermartingale convergence theorem implies either $u'(c_{it})$ is unbounded or it converges to some random variable u'^*

- Could it be unbounded?
  - Only if c_{it} gets arbitrarily close to 0; rule out by assumption that income bounded away from zero (wouldn't be optimal!)
- Could it converge to random variable u'^*?
  - If nonzero: implies convergence of c to positive finite c^*
  - But consumption can't converge if income fluctuates!
  - So u'^* has to be zero, c^* is infinite; **requires infinite assets**!

WHAT WE CONCLUDE

- If $\beta(1+r) \geq 1$, then households tend towards infinite assets and consumption
  - Intuitively: if $\beta(1+r) = 1$ and no uncertainty, constant c
  - Uncertainty and borrowing constraint create upward drift
  - Need $\beta(1+r) < 1$ to cancel this out
  - Otherwise, consumption + assets drift up unboundedly

- We want steady states with finite assets, so assume $\beta(1+r) < 1$
  - more formal proof in Chamberlain and Wilson (2000)

HOW TO SOLVE HOUSEHOLD PROBLEM ON COMPUTER

- Analytical solutions generally don't exist
- Can do value function iteration on Bellman equation
  - speed and accuracy not great
- Better: directly use first-order and envelope conditions

$$u'(c) \geq \beta \mathbb{E}[V_a(e', a') \,|\, e] \qquad V_a(e, a) = (1 + r)u'(c)$$

- Here, these are necessary and sufficient for optimum
  - crucial property: concave objective & convex choice set

- Best way of doing this: "endogenous gridpoints" (Carroll 2006)
  - still backward iteration till convergence, we'll try it soon

DISTRIBUTION OF HOUSEHOLDS

SOLVED HOUSEHOLD PROBLEM, NOW AGGREGATE

- We solved problem facing an individual household
- Generally we'll contemplate economies with a **continuum** of such households, and consider aggregate outcomes
  - soon, put in **general equilibrium**
  - but now, interested in "**partial equilibrium**" properties of model, e.g. total asset demand

- This is a **heterogeneous-agent economy**
  - with a **distribution** of households across the two states, exogenous e and endogenous assets a

HOW DO WE REPRESENT DISTRIBUTION OF HOUSEHOLDS?

- In general, it's a **measure** μ
- If finitely many e, then we can define $\mu(e, \mathbb{A})$ separately for each e, as a measure on subsets $\mathbb{A}$ of the asset space

- Law of motion

$$\mu_{t+1}(e', \mathbb{A}) = \sum_e \mu_t \left(e, (a')^{-1}(e, \mathbb{A})\right) \cdot P(e, e')$$

where $P(e, e')$ is transition probability and $(a')^{-1}(e, \cdot\,)$ is inverse of policy $a'(e, \cdot\,)$

- Measure of $\mathbb{A}$ today is sum of measures yesterday that send you there today

WHY MEASURE?

- You might want some nice density function
  - But will be a positive mass at borrowing constraint (why?)
  - If finitely many e, this leads to a discrete distribution with only mass points (corresponding to distinct histories of e since constraint last binding)
  - Other pathologies possible, could fix with smoother shocks

- In practice on computer: will restrict to distribution on grid
  - Use “lotteries” to make this a better approximation

WHAT IS A STEADY STATE OF THE MODEL?

- Consists of:
  - **policy functions** $a'(e,a)$ and $c(e,a)$ that solve Bellman
  - **measure** $\mu(e,\mathbb{A})$ that satisfies steady-state law of motion

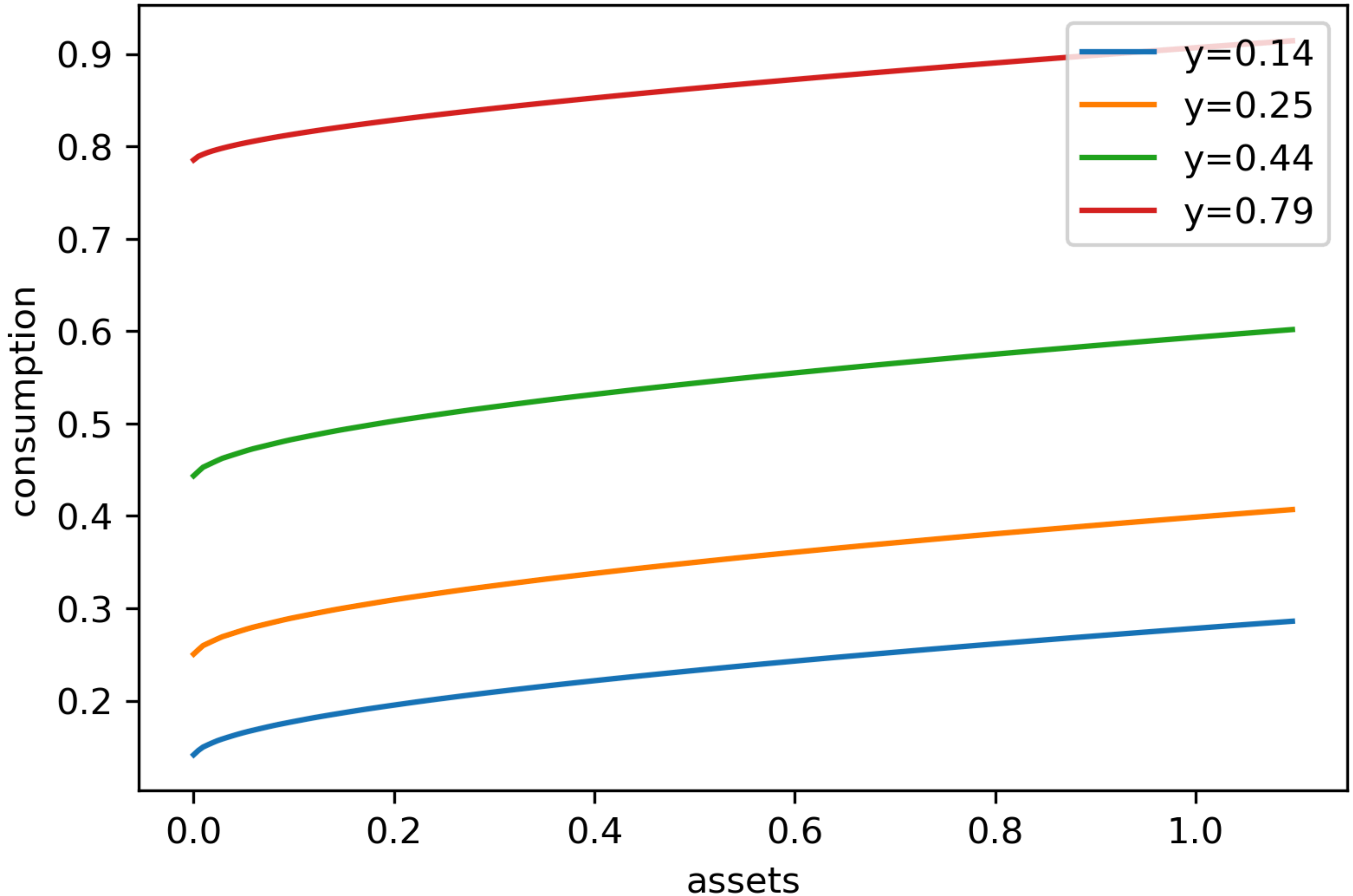
$$\mu(e',\mathbb{A}) = \sum_s \mu\left(e,(a')^{-1}(e,\mathbb{A})\right) \cdot P(e,e')$$

- Can show such a measure exists and is unique if $\beta(1+r) < 1$
  - Why? If finitely many e, then $a'(e,a)$ never leaves some bounded set, contraction argument within this set
- **Aggregate assets and consumption**:

$$A = \int a d\mu = \int a'(e,a) d\mu \qquad C = \int c d\mu$$

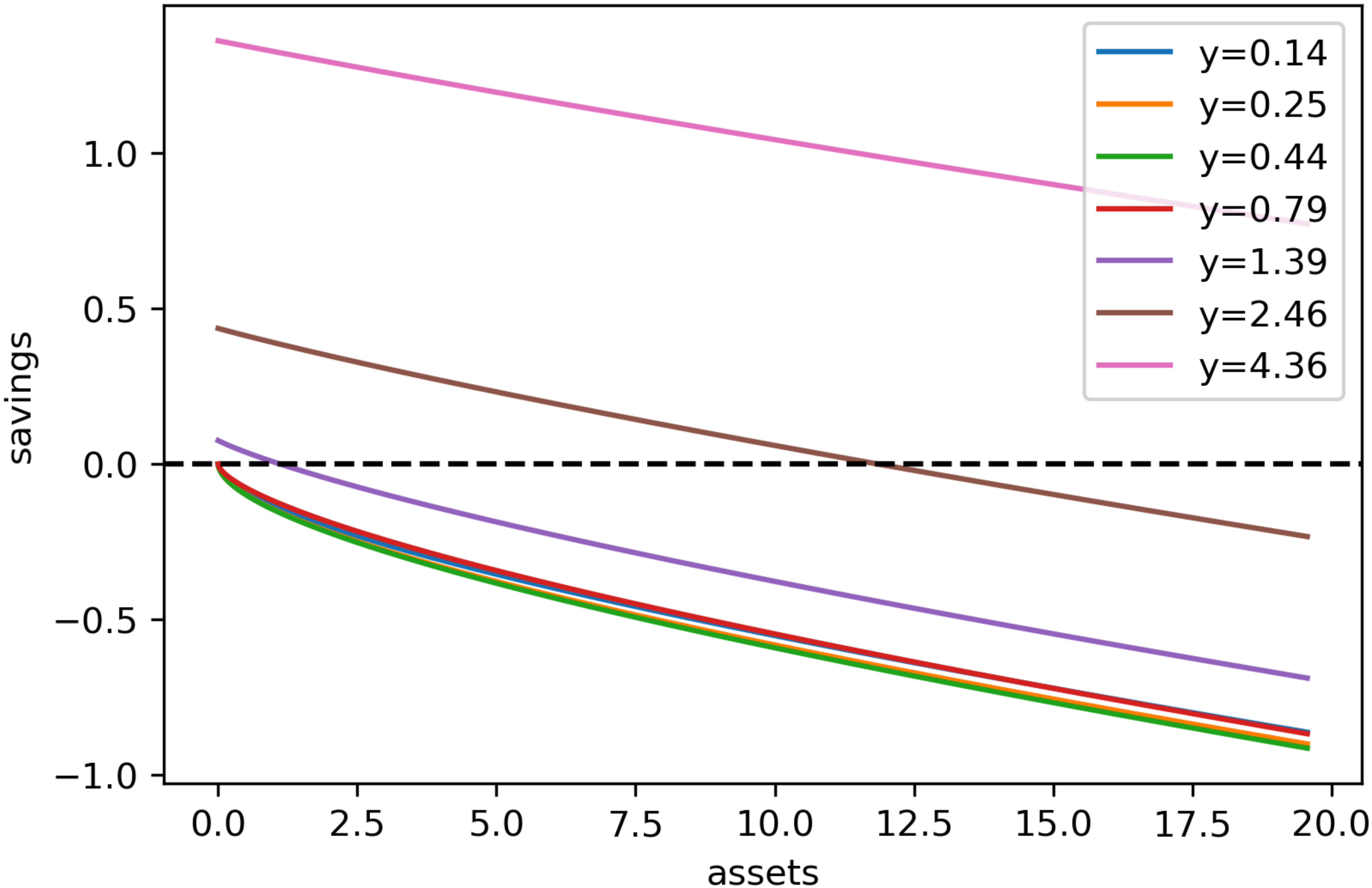
SOME PROPERTIES OF THE MODEL

CONSUMPTION INCREASING AND CONCAVE IN ASSETS

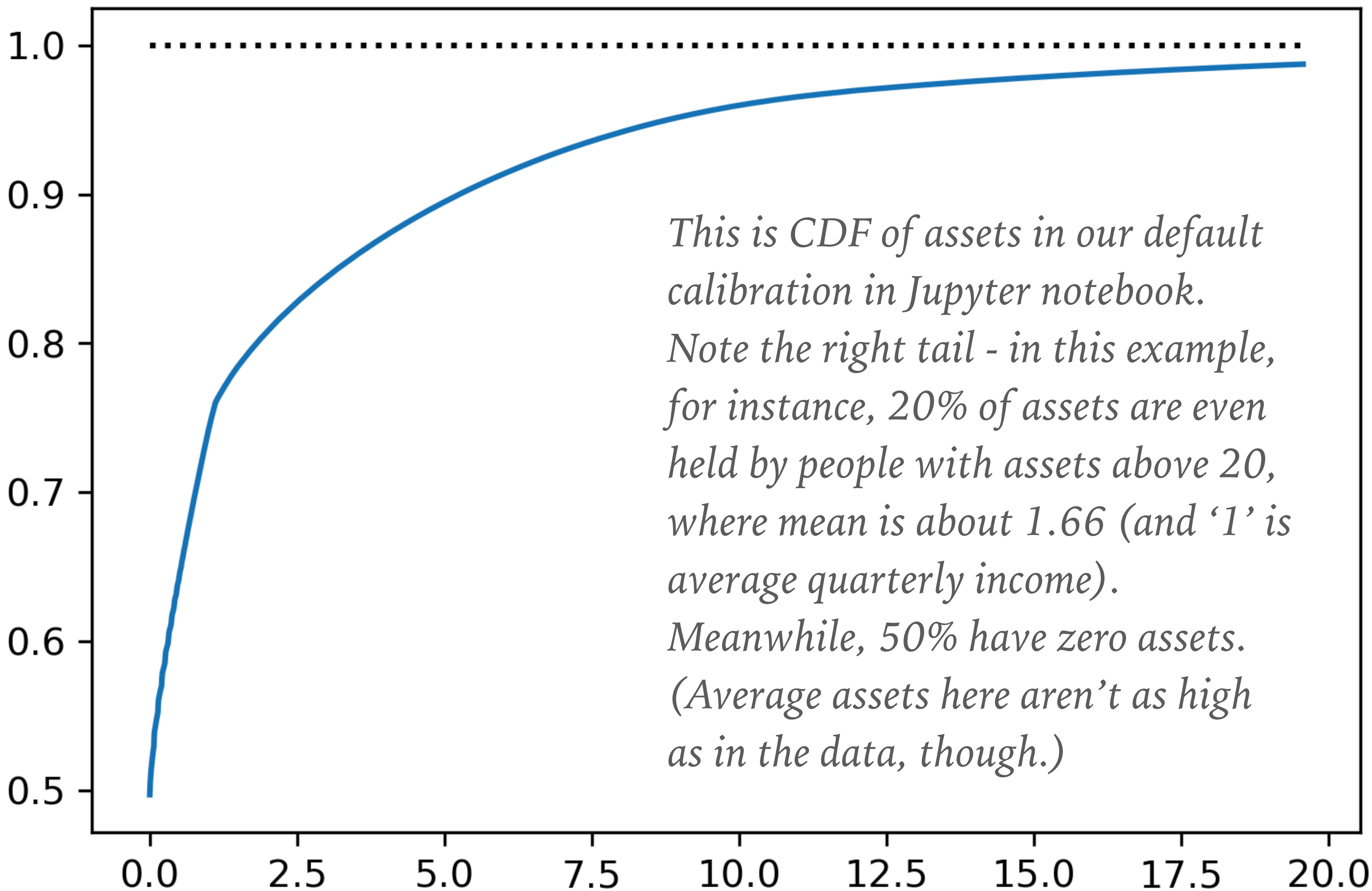


(generally increasing in income too, but need more structure on Markov chain to be sure)

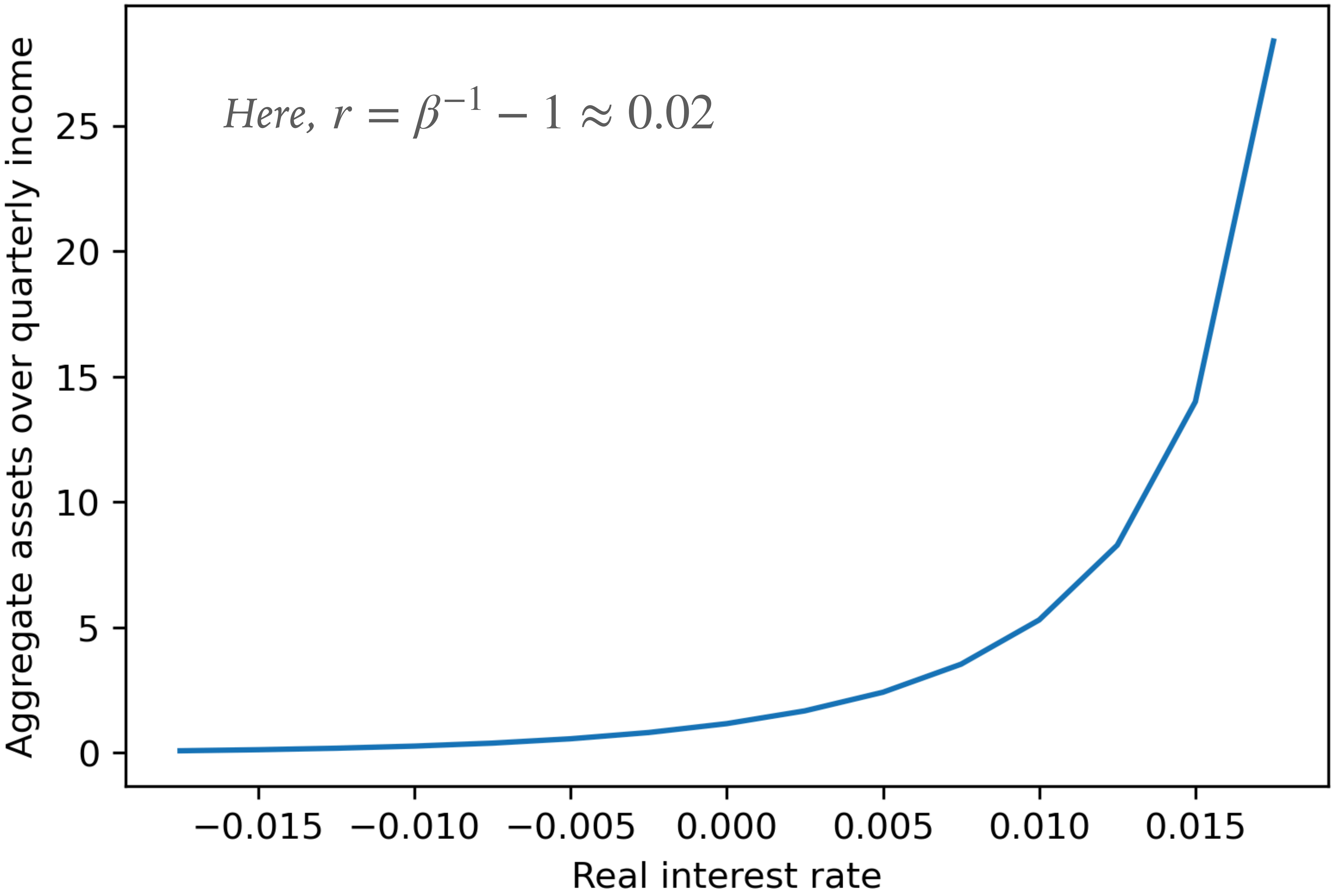
SAVINGS (INCOME MINUS CONSUMPTION) DECREASING IN ASSETS



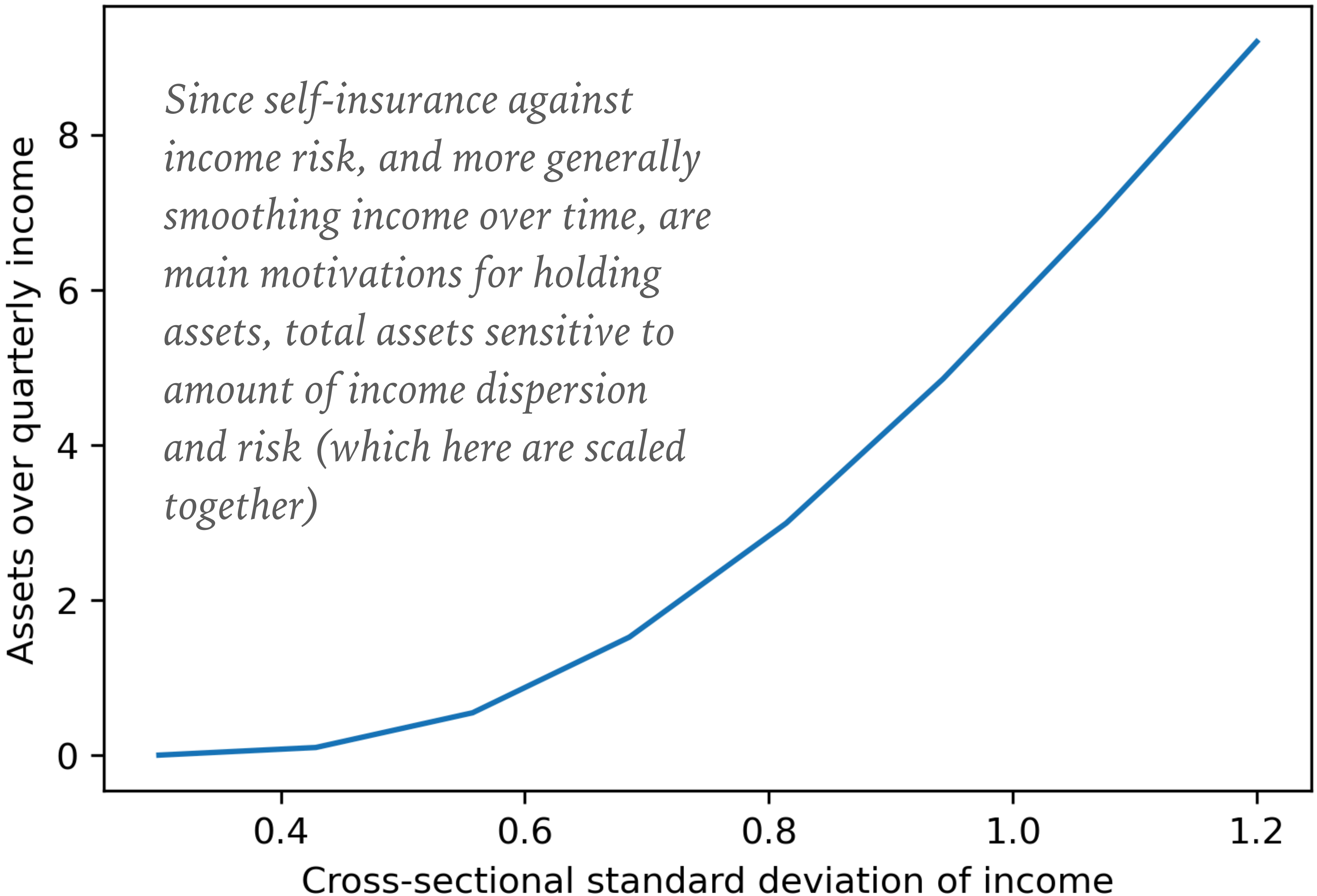
USUALLY A DECENT AMOUNT OF INEQUALITY IN ASSET HOLDINGS



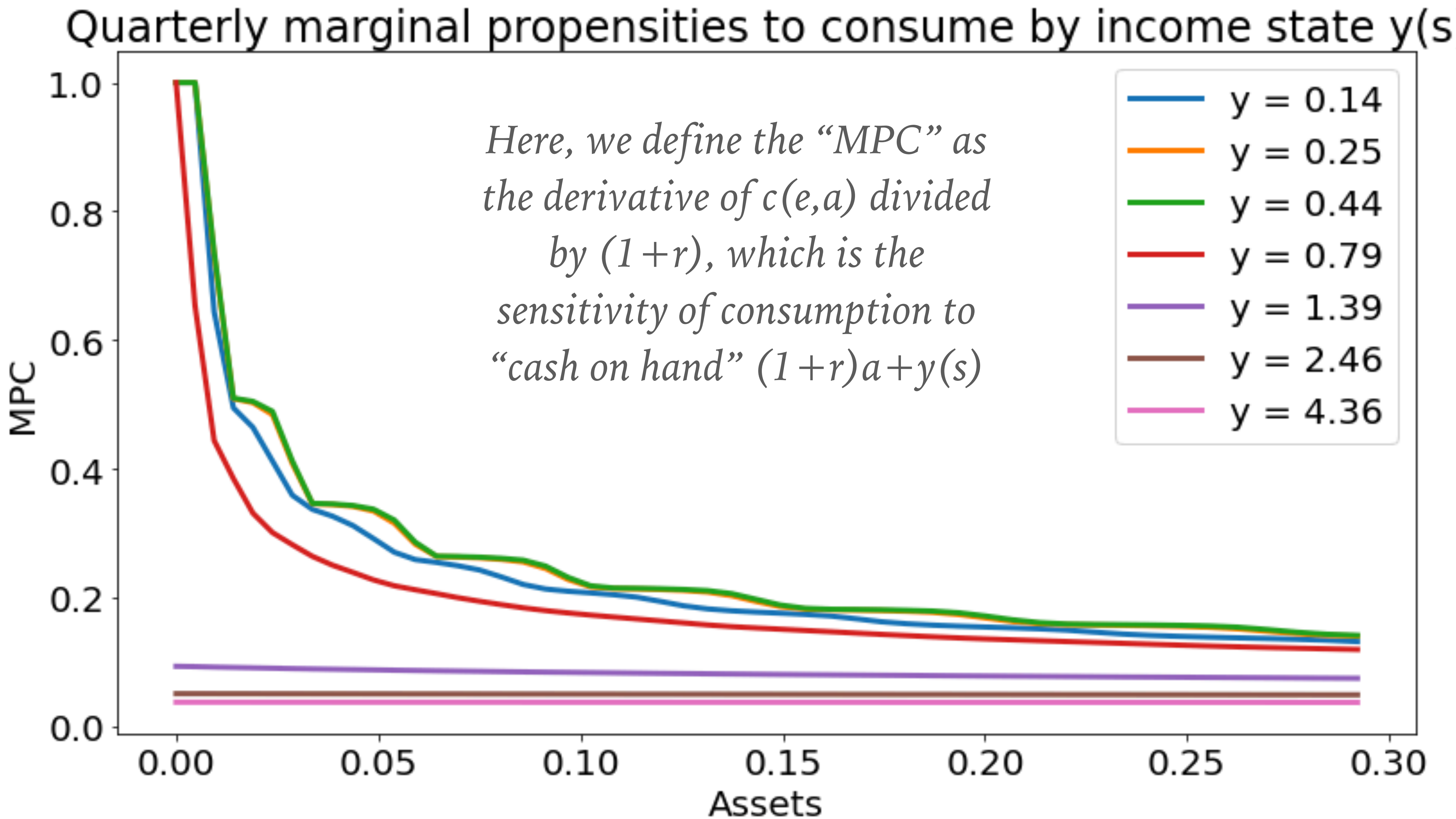
AS ANALYSIS SUGGESTED, ASSETS ASYMPTOTE AT $\beta(1+r) = 1$



MORE ASSETS WHEN INEQUALITY / RISK ARE HIGHER



CONCAVE CONSUMPTION IMPLIES DECLINING MPCS



Here, we define the "MPC" as the derivative of c(e,a) divided by (1+r), which is the sensitivity of consumption to "cash on hand" (1+r)a+y(s)

GREAT FEATURES OF THE MODEL

- Fixed many complaints about representative-agent model
  - don't need $\beta(1 + r) = 1$; elasticity of asset accumulation is not infinity
  - can potentially match higher MPCs
  - endogenously generates wealth inequality
  - all reasons this is a very nice core model for macro!

- Will turn out to be imperfect on all these dimensions
  - elasticity of assets often still too high, MPCs too low, when we match high aggregate wealth in data, can't generate enough wealth inequality in the tail

QUANTITATIVE LIMITATIONS OF THE MODEL

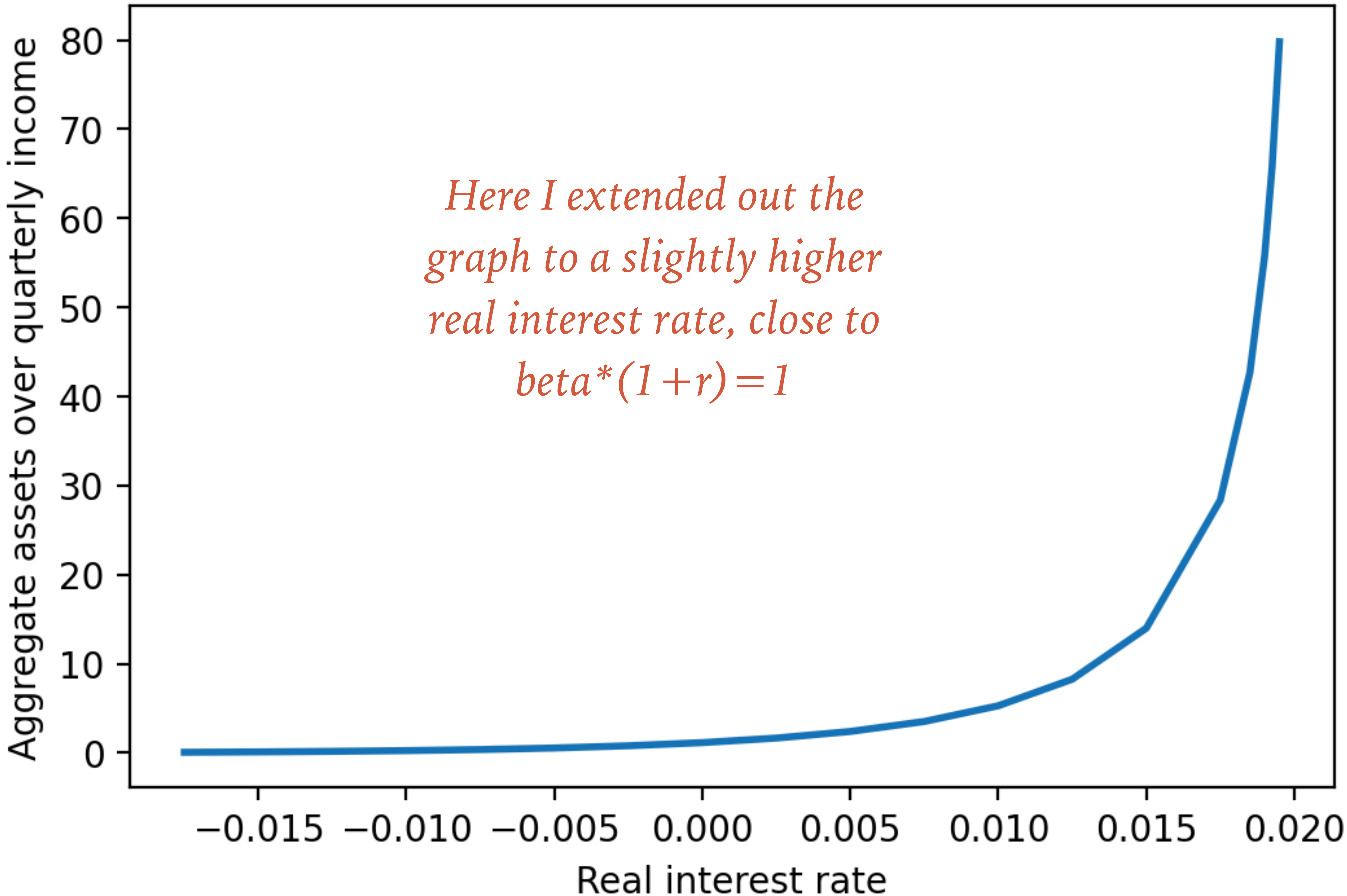
THREE ADVANTAGES, THREE LIMITATIONS

- Elasticity of asset accumulation is **not infinity**
  - but it's still **very very high**!

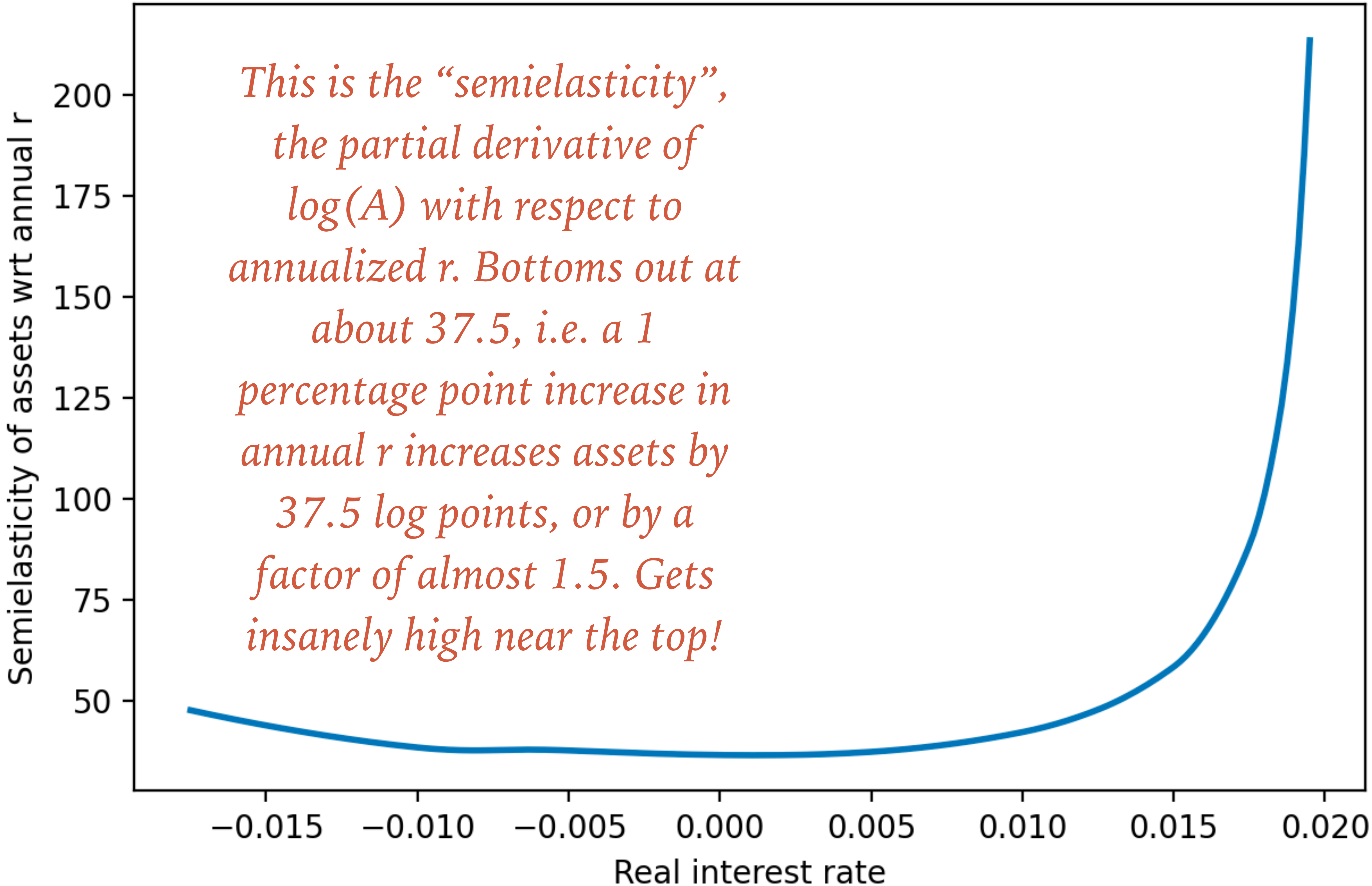
- MPCs are well above rep agent, closer to empirically relevant
  - but this is hard to match in a calibration with high wealth!

- Endogenously get a lot of wealth inequality
  - a pretty accurate amount, actually, but the "tail" is too thin!

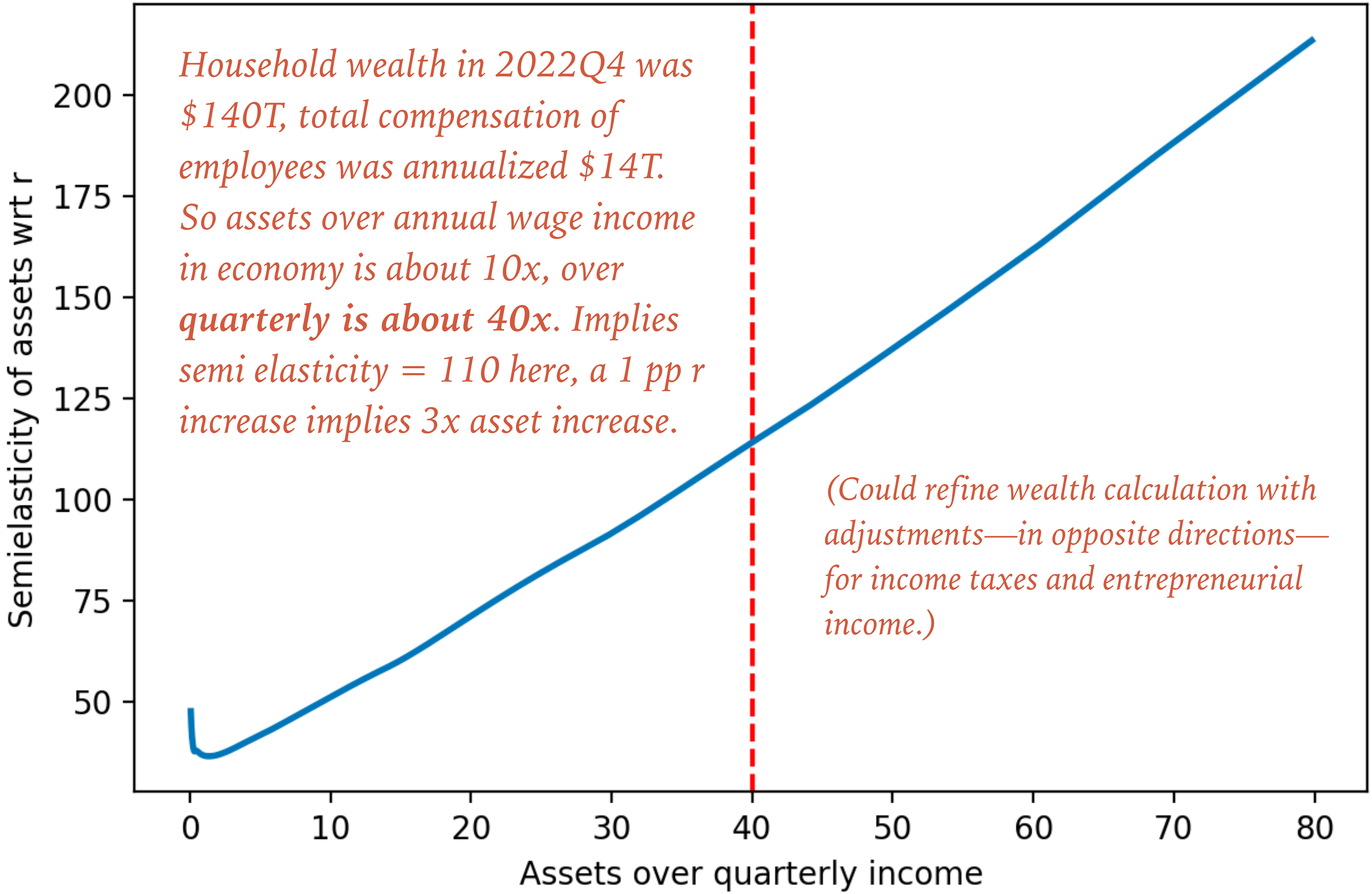
ASYMPTOTE IS REALLY EXTREME



AND STEADY-STATE ASSETS CAN BE VERY SENSITIVE TO R



BUT REALLY BAD IF WE MATCH WEALTH

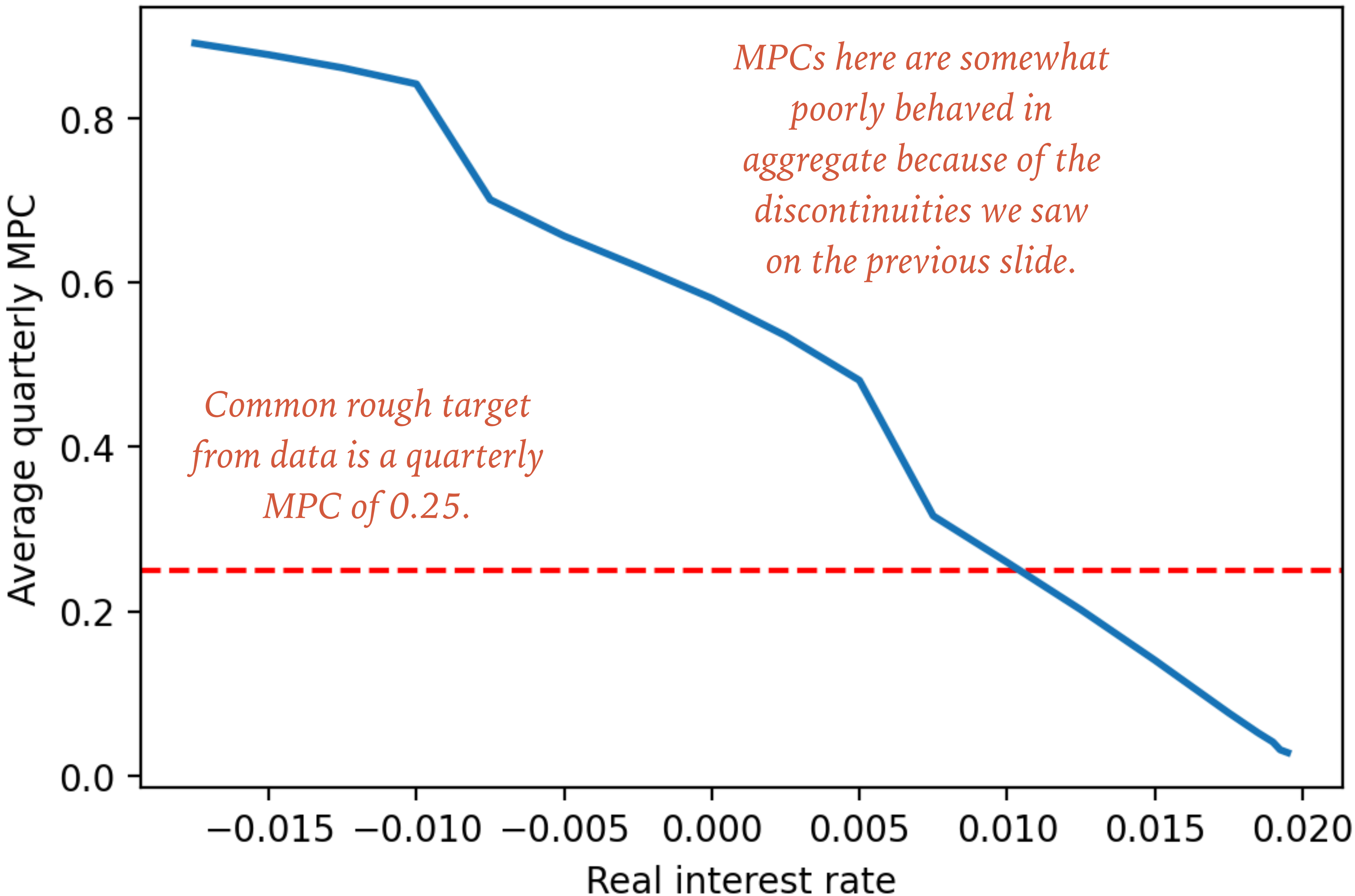


WHAT IS A MORE REASONABLE SEMIELASTICITY?

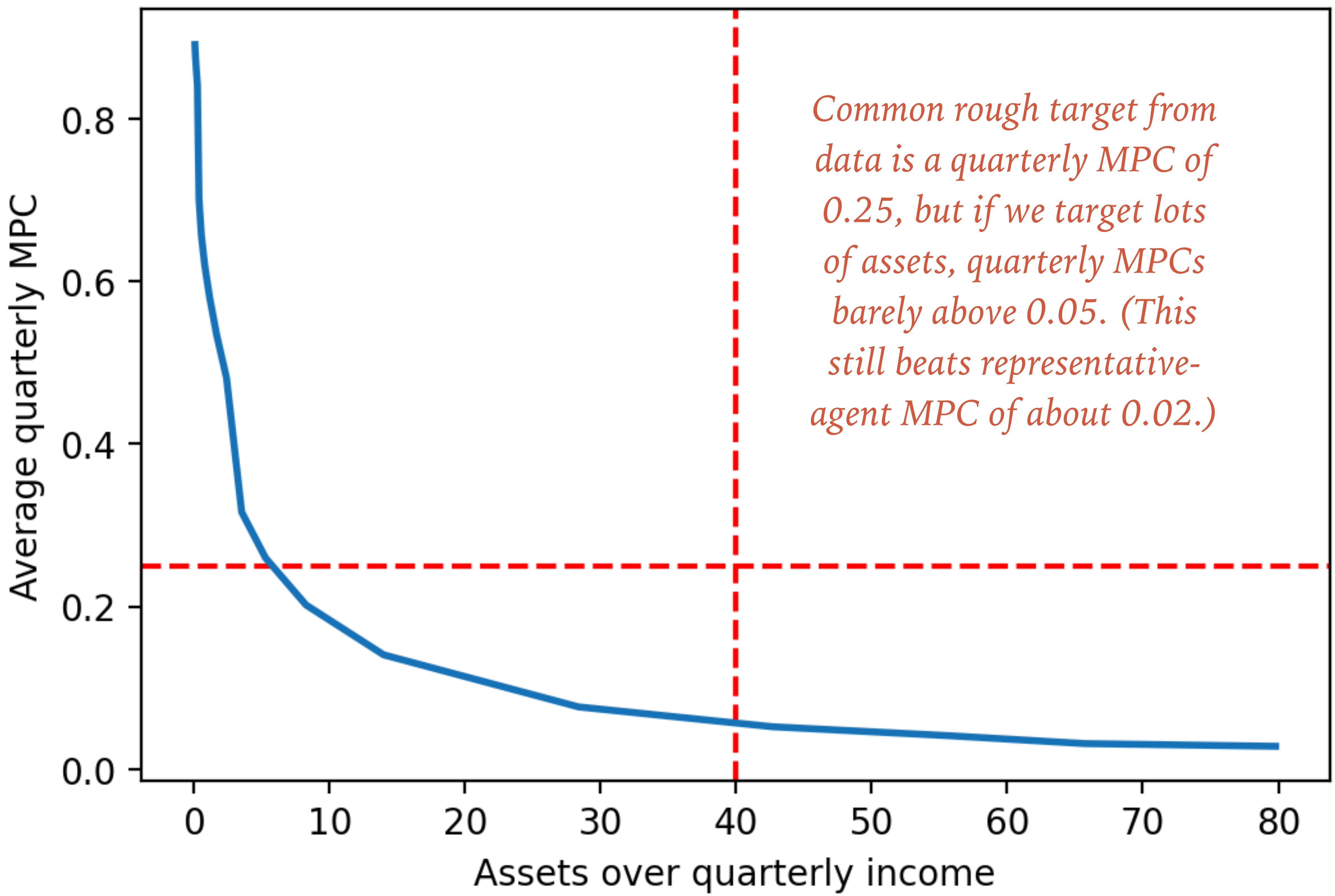
- Moll, Rachel, and Restrepo (2021) survey empirical literature and identify range for semi-elasticity of **1.25 to 35**.
  - We just found **110** here when calibrating wealth!
  - Some changes could help (mostly if we pick a lower elasticity of intertemporal substitution), but huge gulf.

- Problem: income smoothing not a strong enough motive to hold lots of wealth, so wealth becomes very rate-sensitive at asymptote, not much different from representative agent.
- Life-cycle model of Auclert, Malmberg, Martenet, Rognlie: benchmark of about **37.5**.

CAN MATCH HIGH MPCS, BUT NOT CLOSE TO ASYMPTOTE



WHAT IF WE TRY TO MATCH ASSETS AGAIN?



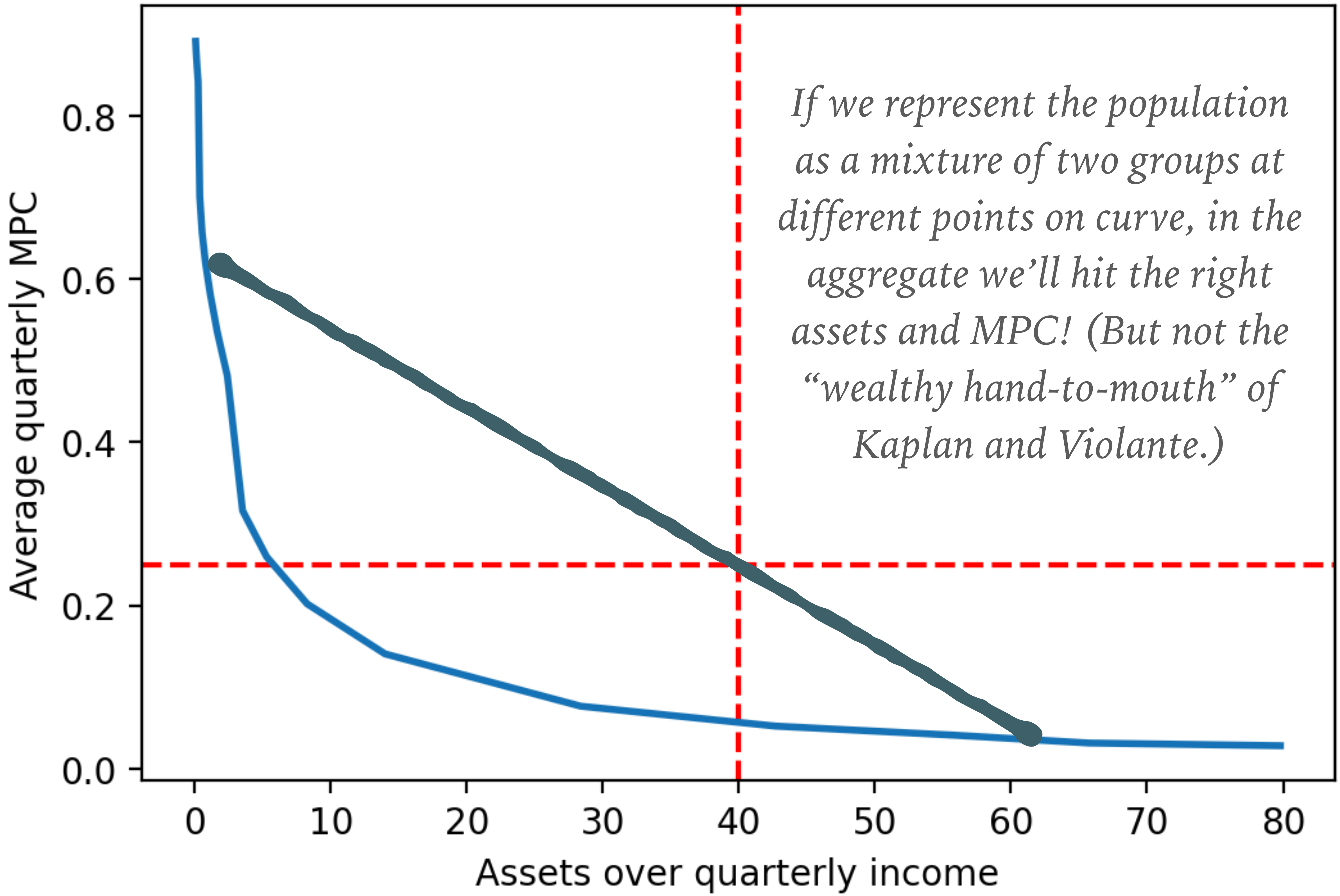
HOW DO WE SOLVE THIS PUZZLE?

- One idea: more heterogeneity
  - e.g. Carroll, Slacalek, Tokuoka, White (2017)
  - Some people are patient and save a lot, holding most of the wealth and having low MPCs, others the opposite

- Other idea: not all wealth is "liquid"
  - e.g. Kaplan, Violante, Weidner (2014)
  - Houses, retirement accounts, small businesses, etc. are big part of wealth but can't be used to smooth income

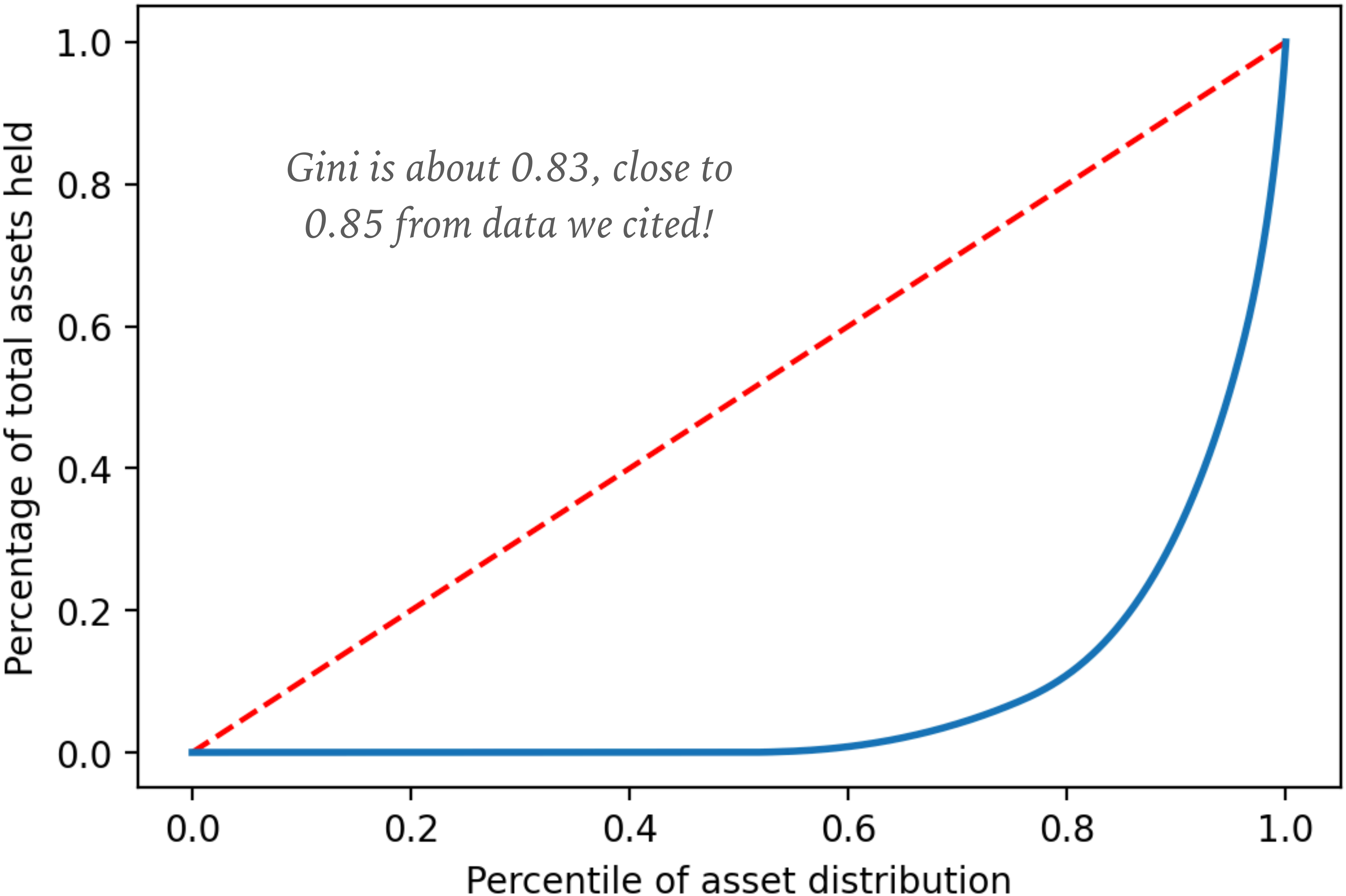
- Same as before: more motives for saving would help
  - Problem is with a lot of wealth, few people are close to zero assets

ROUGH INTUITION BEHIND HETEROGENEITY IDEA

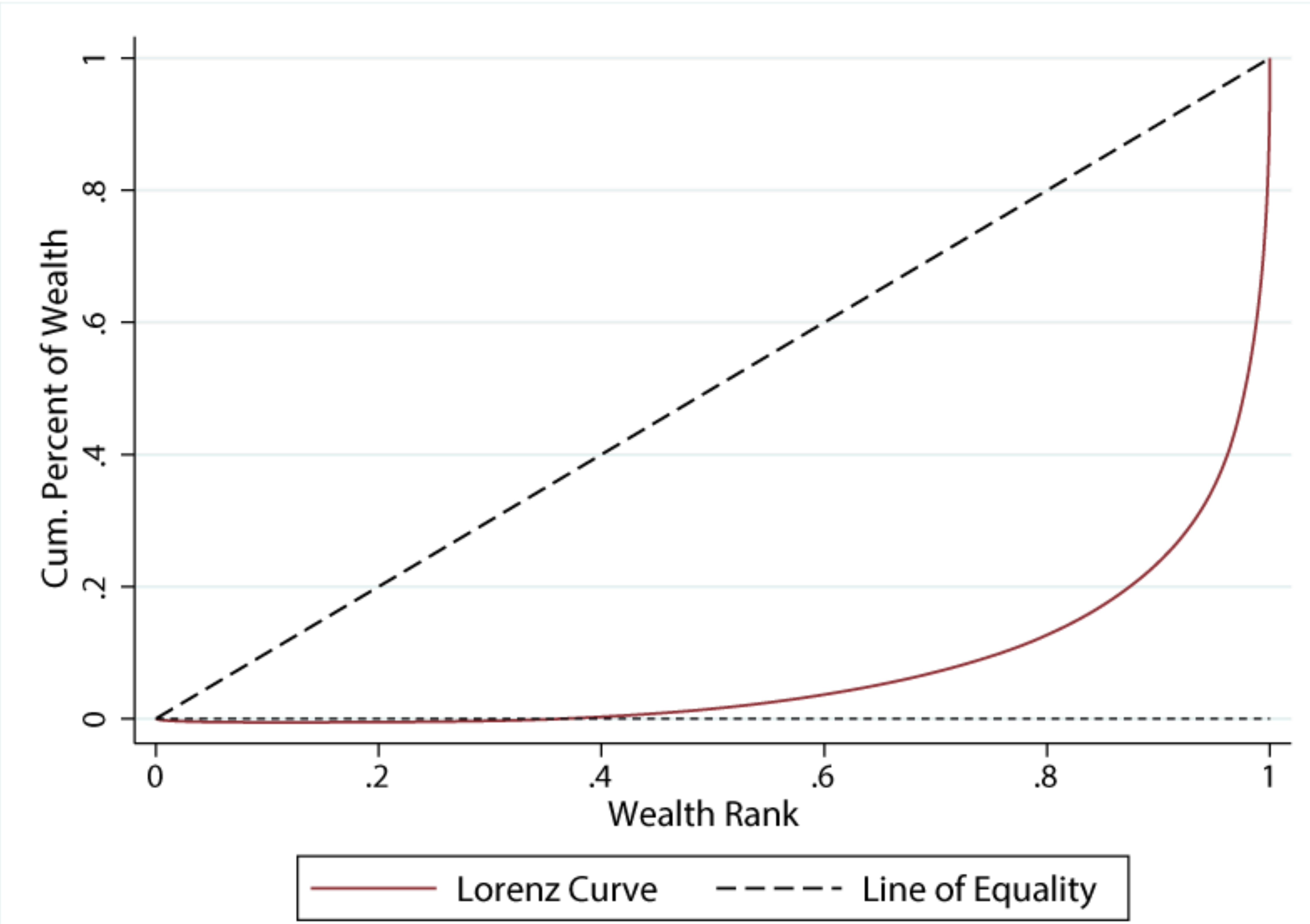


If we represent the population as a mixture of two groups at different points on curve, in the aggregate we'll hit the right assets and MPC! (But not the "wealthy hand-to-mouth" of Kaplan and Violante.)

WE GET A DECENT-LOOKING LORENZ CURVE IN BASELINE CALIBRATION



COMPARE TO PREVIOUS IN THE DATA…



BUT THERE ARE SUBTLE ISSUES!

- The model says that the “middle class” hold too few assets
  - while missing the assets held by the extreme rich!
  - these roughly offset each other for Gini coefficient, but important misses

- Benhabib, Bisin, Luo (2017): in this model, thickness of the tail of wealth is given by the thickness of the tail of income
  - Both have a Pareto tail in practice, but wealth is a “fatter” Pareto distribution, which this model can’t explain
  - Need other ingredients to fix this!