INTRODUCTION TO THE CANONICAL HANK MODEL AND FISCAL POLICY

Econ 411-3

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REMINDER: STANDARD INCOMPLETE MARKETS MODEL

$$V_t(e,a) = \max_{c,a'} u(c) + \beta \mathbb{E}[V_{t+1}(e',a') \mid e]$$

$$s.t.\ a' + c = (1 + r_t)a + y_t(e)$$

$$a' \geq \underline{a}$$

Time-varying distribution

$$\mu_t(e,a)$$

Aggregate outcomes, e.g.

$$A_t = \int a_t(e,a) d\mu_t(e,a)$$

START WITH BEWLEY MODEL

- Bewley GE version of standard incomplete markets has:
  - aggregate household assets = government bonds
  - exogenous labor (as in usual SIM) produces good, real interest rate adjusts to clear the market
  - government levies taxes to pay interest

- This model embeds SIM with:

$$A_t = B_t \qquad y_t(s) = (Z_t - \tau_t)e \qquad r_t = r_{t-1}^{ante} \qquad Y_t = Z_t \int e$$

- where r_{t-1}^{ante} is determined endogenously to clear market

A FEW MODIFICATIONS TO GET A CANONICAL "HANK" MODEL

- Assume endowment e gives "skill" per hour N of labor worked, where N is endogenous
  - households have disutility from higher N

- Assume that there is nominal wage rigidity, and that labor margin is not flexible, so that at any given moment, households are "rationed" into working however much of their labor N is determined at the posted wage
  - full model: Phillips curve where wages adjust
  - what we'll do now: **nominal wages perfectly rigid** (won't change any real outcomes in our basic model)

MODIFICATIONS CONTINUED

- Assume no productivity shocks Z_t for now, i.e. $Z_t = 1$
- Then $P_t = W_t$, real wages $w_t = W_t/P_t = 1$
- Assume central bank sets **nominal interest rate** exogenously
  - which equals **real interest rate** because inflation = 0
- We have a model where

$$A_t = B_t \quad y_t(s) = (1 - \tau_t)N_t e \qquad r_t = r_{t-1}^{ante} \quad Y_t = N_t \int e$$

- … and r_t^{ante} set exogenously by central bank

ALSO ADD GOVERNMENT SPENDING

- Suppose government has spending G on final goods:
  - Market clearing: $$Y_t = C_t + G_t$$
  - Government budget: $$\tau_t N_t \int e + B_t = G_t + (1 + r_{t-1}^{ante}) B_{t-1}$$

- Simplifying assumption: no direct effect of G on households or production (either it's useless or enters utility separately)

- Assume $\int e = 1$, define $T_t \equiv \tau_t N_t$, use $Y_t = N_t$, then have:

$$y_t(s) = (Y_t - T_t)e \qquad A_t = B_t$$

$$B_t - (1 + r_{t-1}^{ante}) B_{t-1} = G_t - T_t \qquad Y_t = C_t + G_t$$

These four equations, plus the household side, characterize equilibrium!

SUM UP OUR ECONOMY: SIMPLE LAB FOR FISCAL & MONETARY POLICY

- Monetary authority exogenously sets path of r_t^{ante}
- Fiscal authority chooses paths of tax revenue T_t, government spending G_t, and bonds B_t, subject to flow budget constraint

$$B_t - (1 + r_{t-1}^{ante})B_{t-1} = G_t - T_t$$

- Households behave as in the standard incomplete markets model, given income $y_t(e) = (Y_t - T_t)e$ and $r_t = r_{t-1}^{ante}$, with their aggregate behavior determining C_t and A_t
- Goods and asset markets clear:

$$Y_t = C_t + G_t \qquad A_t = B_t$$

Notes:

1. *We'll generally assume bounded paths for all policy variables*
2. *By Walras's law, either goods or asset market clearing is redundant*

EQUILIBRIUM TAKING POLICY AS GIVEN

- Define consumption and asset functions

$$\mathcal{C}_t(\{Y_s - T_s\}, \{r_s^{ante}\}) \qquad \mathcal{A}_t(\{Y_s - T_s\}, \{r_s^{ante}\})$$

  which give aggregate household consumption and assets at each date t as a function of paths of aggregate after-tax income and real interest rates at all dates s (since these determine all inputs to household problem)

- Then, taking policy $\{G_s, T_s, B_s, r_s^{ante}\}$as given, sum up equilibrium in just one series of equations, either goods or asset space:

$$Y_t = \mathcal{C}_t(\{Y_s - T_s\}, \{r_s^{ante}\}) + G_t$$

$$B_t = \mathcal{A}_t(\{Y_s - T_s\}, \{r_s^{ante}\})$$

GOODS MARKET CLEARING: AN INTERTEMPORAL KEYNESIAN CROSS

- Assume monetary policy holds r constant for now (so we can ignore that input), and think about goods market clearing:

$$Y_t = \mathscr{C}_t(\{Y_s - T_s\}) + G_t$$

- Define derivatives $M_{ts} \equiv \partial \mathscr{C}_t / \partial (Y_s - T_s)$
  - these form the Jacobian matrix $\mathbf{M}$ of aggregate C_t vs. $Y_s - T_s$
  - Auclert, Rognlie, Straub (2024): intertemporal MPCs
- Then to first order, stacking in vectors, equation becomes

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

an "intertemporal Keynesian cross" (IKC)

INTERTEMPORAL VS. TRADITIONAL KEYNESIAN CROSS

- Intertemporal Keynesian cross

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

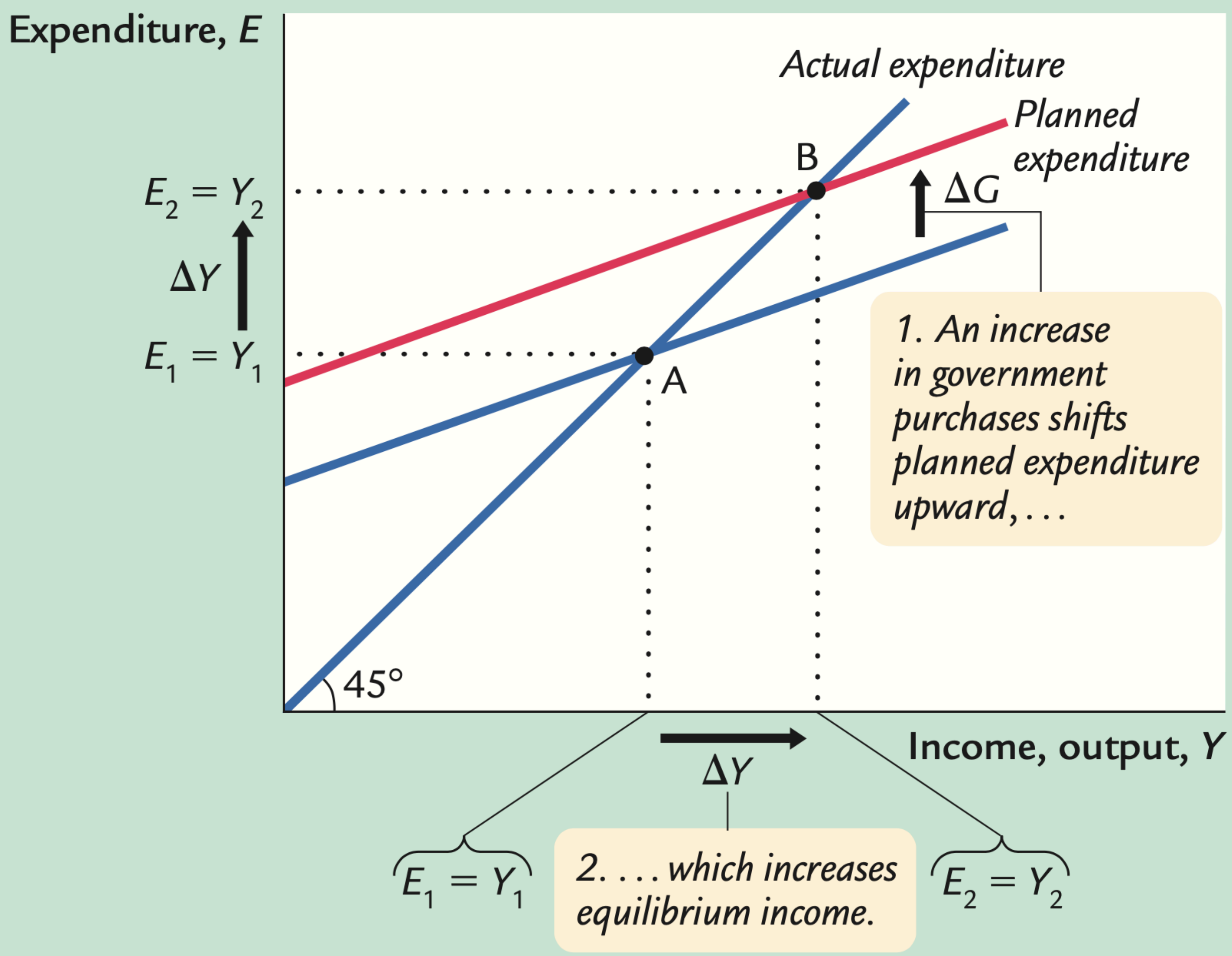
  - income feeds back through consumption to itself via $\mathbf{M}$
  - entire complexity of model is in $\mathbf{M}$
  - in principle, if we knew $\mathbf{M}$ from the data, could get $d\mathbf{Y}$ without model (in practice, not enough data)

- Static, traditional Keynesian cross, with "mpc" a scalar:

$$dy = dg - mpc \cdot dt + mpc \cdot dy \longrightarrow dy = \frac{dg - mpc \cdot dt}{1 - mpc}$$

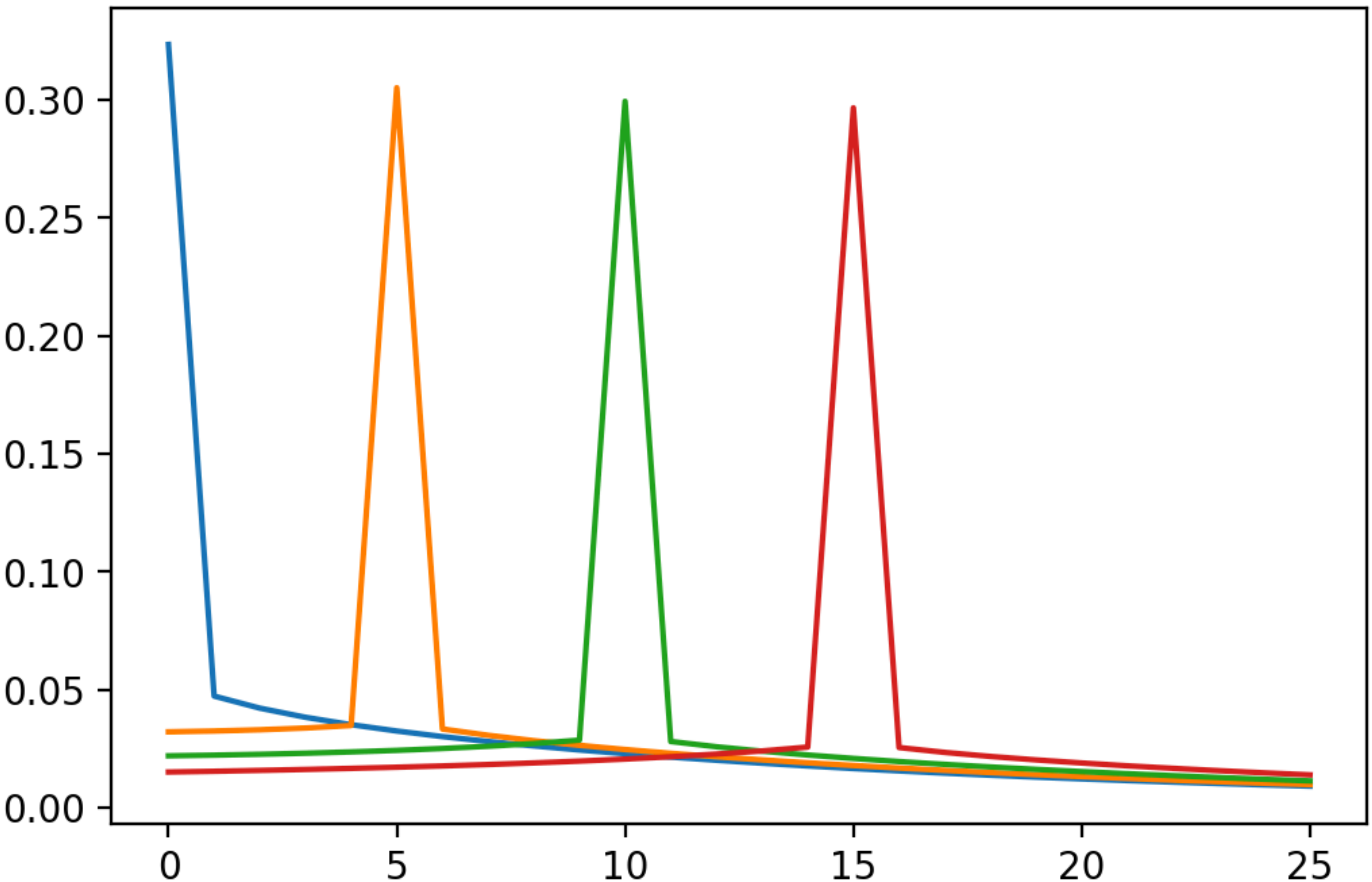
BRINGS BACK MEMORIES FROM UNDERGRAD MACRO…

figure 10-5



An Increase in Government Purchases in the Keynesian Cross An increase in government purchases of ΔG raises planned expenditure by that amount for any given level of income. The equilibrium moves from point A to point B, and income rises from Y_1 to Y_2. Note that the increase in income ΔY exceeds the increase in government purchases ΔG. Thus, fiscal policy has a multiplied effect on income.

WHAT DO INTERTEMPORAL MPCS LOOK LIKE? AN EXAMPLE CALIBRATION



CAN WE GET MULTIPLIER FOR INTERTEMPORAL KEYNESIAN CROSS?

- Static Keynesian cross solved like

$$dy = dg - mpc \cdot dt + mpc \cdot dy \longrightarrow dy = \frac{dg - mpc \cdot dt}{1 - mpc}$$

- Would love to do this for intertemporal Keynesian cross, i.e.

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \longrightarrow d\mathbf{Y} = (\mathbf{I} - \mathbf{M})^{-1}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

- Unfortunately $(\mathbf{I} - \mathbf{M})^{-1}$ doesn't exist, since $\mathbf{I} - \mathbf{M}$ is singular
  - (all income is eventually spent, so PDV of rows = 0)
- Still, usually, unique solution mapping $\mathscr{M}$ (see next slide):

$$d\mathbf{Y} = \mathscr{M}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

HOW DO WE GET SOLUTIONS IN PRACTICE?

- A few ways:
  - some analytical cases (we'll introduce soon) will be so simple that we can derive solution with pen-and-paper
  - defining $\mathbf{A}$ with entries $A_{ts} \equiv \partial \mathscr{A}_t / \partial (Y_s - T_s)$, there is a simple closed form for $\mathscr{M}$:

$$\mathscr{M} = \mathbf{A}^{-1}\mathbf{K}$$

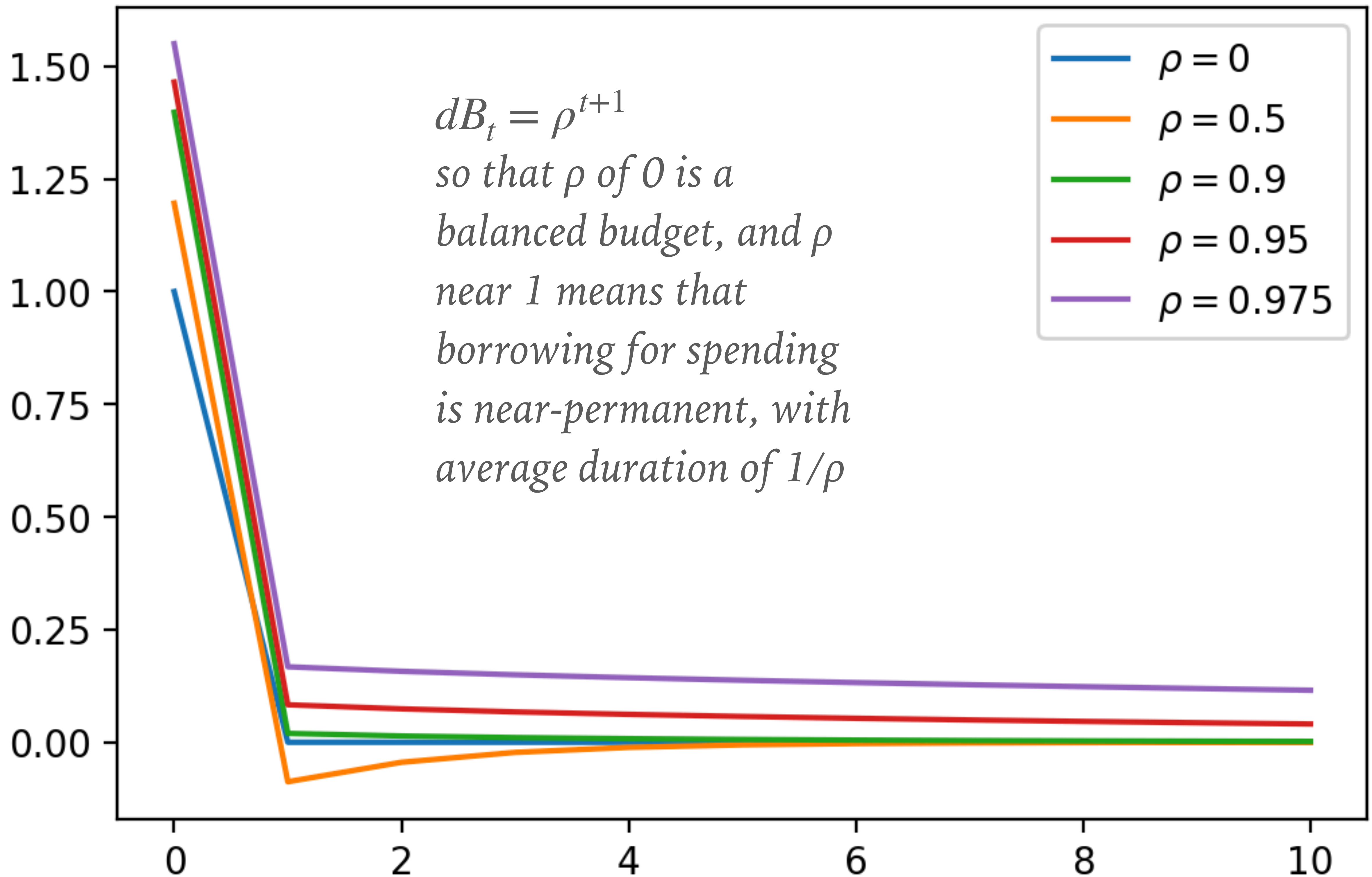
for upper-triangular $K_{ts} = -(1+r)^{t-s}$ for $t \le s$, $K_{ts} = 0$ for $t > s$

  - also can solve in "asset space":

$$B_t = \mathscr{A}_t(\{Y_s - T_s\}) \longrightarrow d\mathbf{B} = \mathbf{A}(d\mathbf{Y} - d\mathbf{T})$$

$$\longrightarrow d\mathbf{Y} = \mathbf{A}^{-1}d\mathbf{B} + d\mathbf{T}$$

EXPERIMENT: SPENDING AT DATE 0, FINANCED PARTLY BY DEBT



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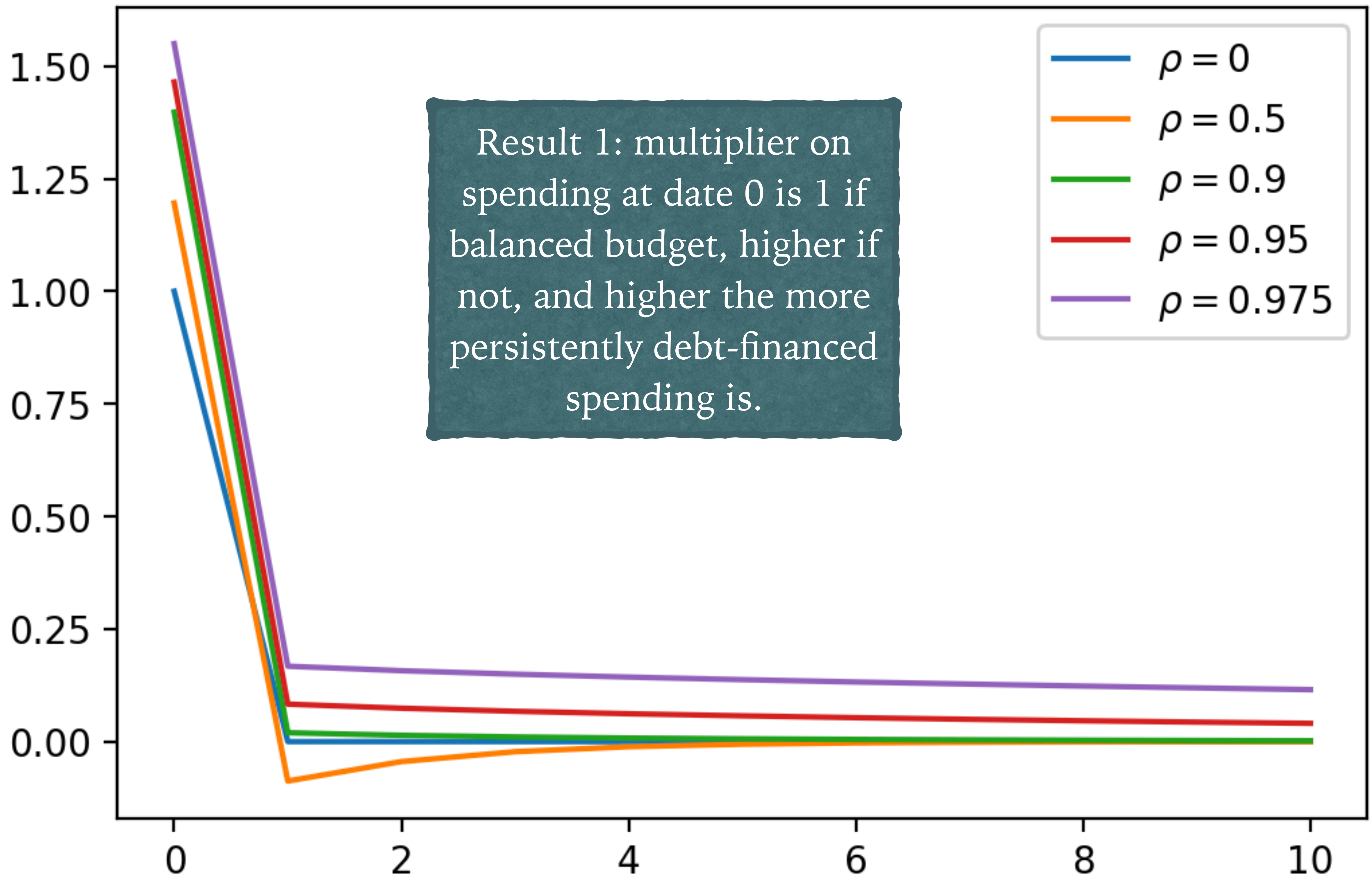

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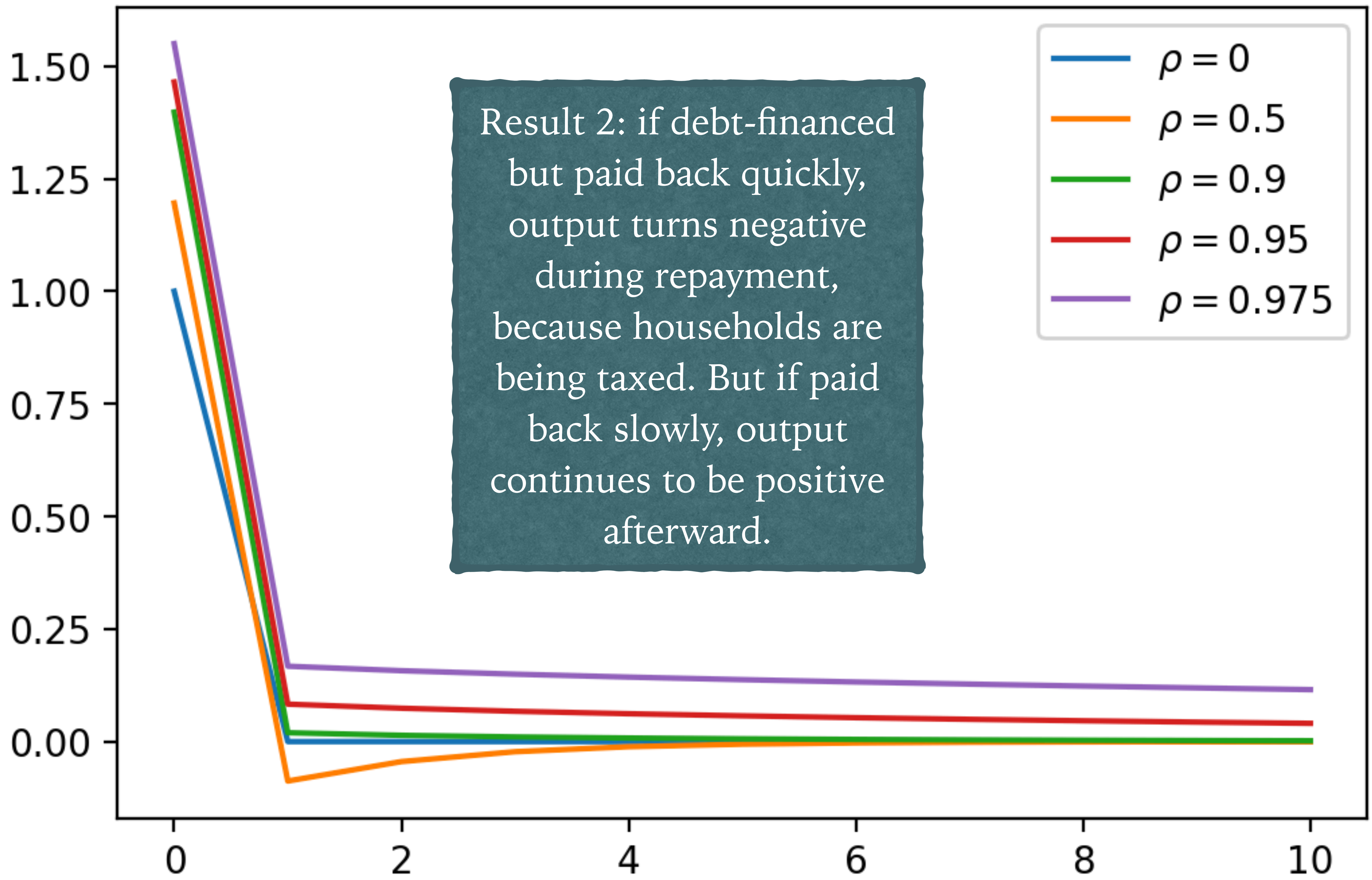


UNDERSTANDING WHAT'S GOING ON

$$dY = dG - MdT + MdY$$

All else equal, higher spending increases output

Especially when taxes are not contemporaneous (so the spending effect felt in other periods)

This higher output dY feeds back into into consumption via MdY, which feeds back into output, and so on. If there is a large positive effect from dG but limited negative effect from taxes -MdT, this amplification causes a large increase in debt

ALTERNATIVE INTUITION FROM ASSET SPACE (ALSO VALID!)

$$d\mathbf{B} = \mathbf{A}(d\mathbf{Y} - d\mathbf{T})$$

In equilibrium, after-tax income $d\mathbf{Y} - d\mathbf{T}$ has to change in such a way that households hold the increase in bonds $d\mathbf{B}$ outstanding. This will generally involve an increase in after-tax income before the bonds are retired, so that households are willing to hold the extra bonds in order to smooth this income.

WHY IS THE MULTIPLIER 1 (WITH NO LAGGED EFFECT) IF BALANCED BUDGET?

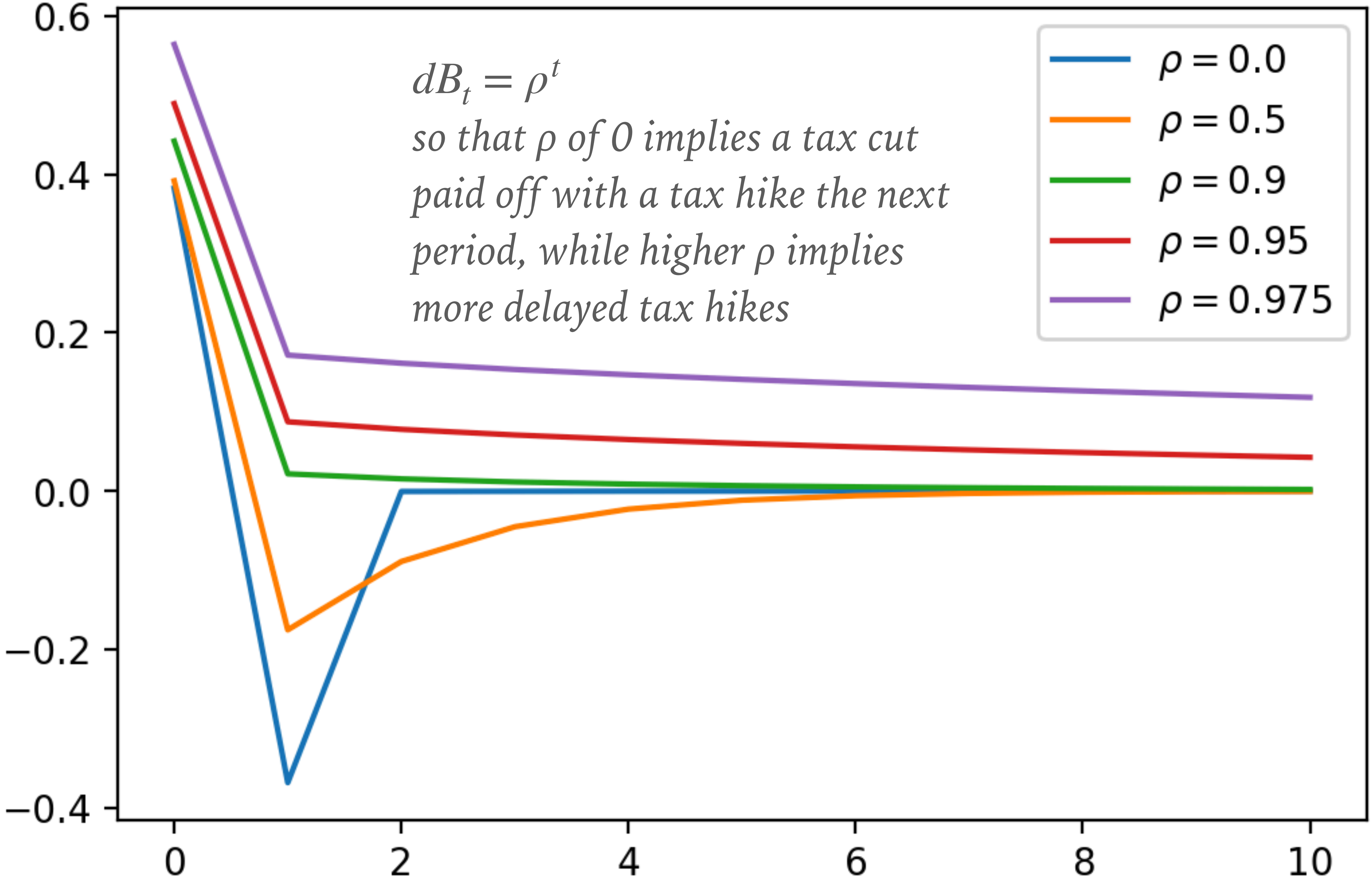
- If balanced budget, then $d\mathbf{G} = d\mathbf{T}$. Conjecture that $d\mathbf{Y} = d\mathbf{G}$ and verify that IKC holds:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \longleftrightarrow \cancel{d\mathbf{Y}} = \cancel{d\mathbf{Y}} - \cancel{\mathbf{M}d\mathbf{Y}} + \cancel{\mathbf{M}d\mathbf{Y}}$$

- Intuitively: the after-tax income earned by households doesn't change, since output increases by exactly the amount of spending, and then exactly this additional income is taxed!

- This requires our assumption that both marginal taxes and labor affect everyone's income proportionally; if they affect income differently (i.e. taxes hit poor), then things would be different

SIMILAR EXPERIMENT FOR DEFICIT-FINANCED TAX CUT AT DATE 0



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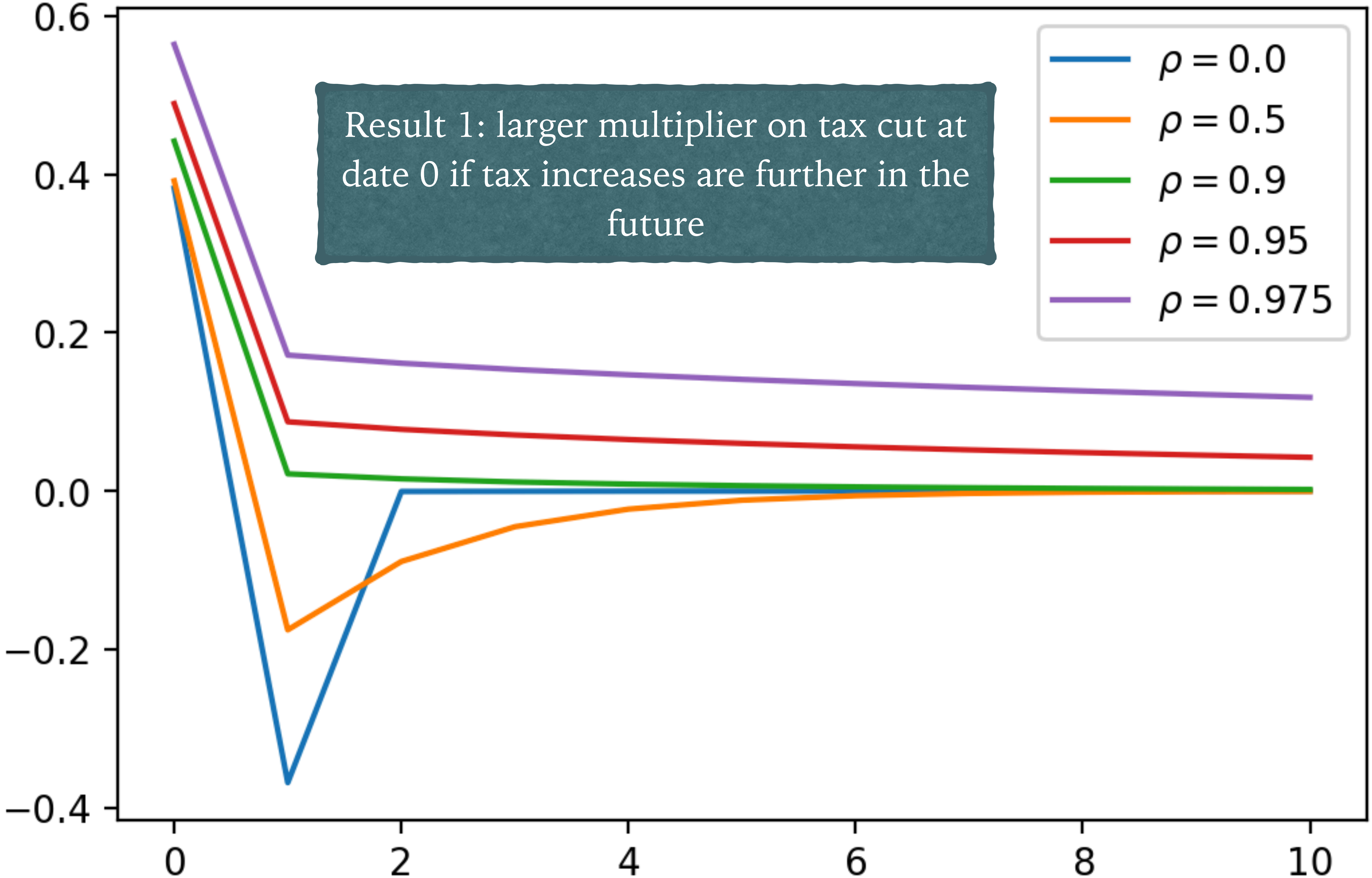

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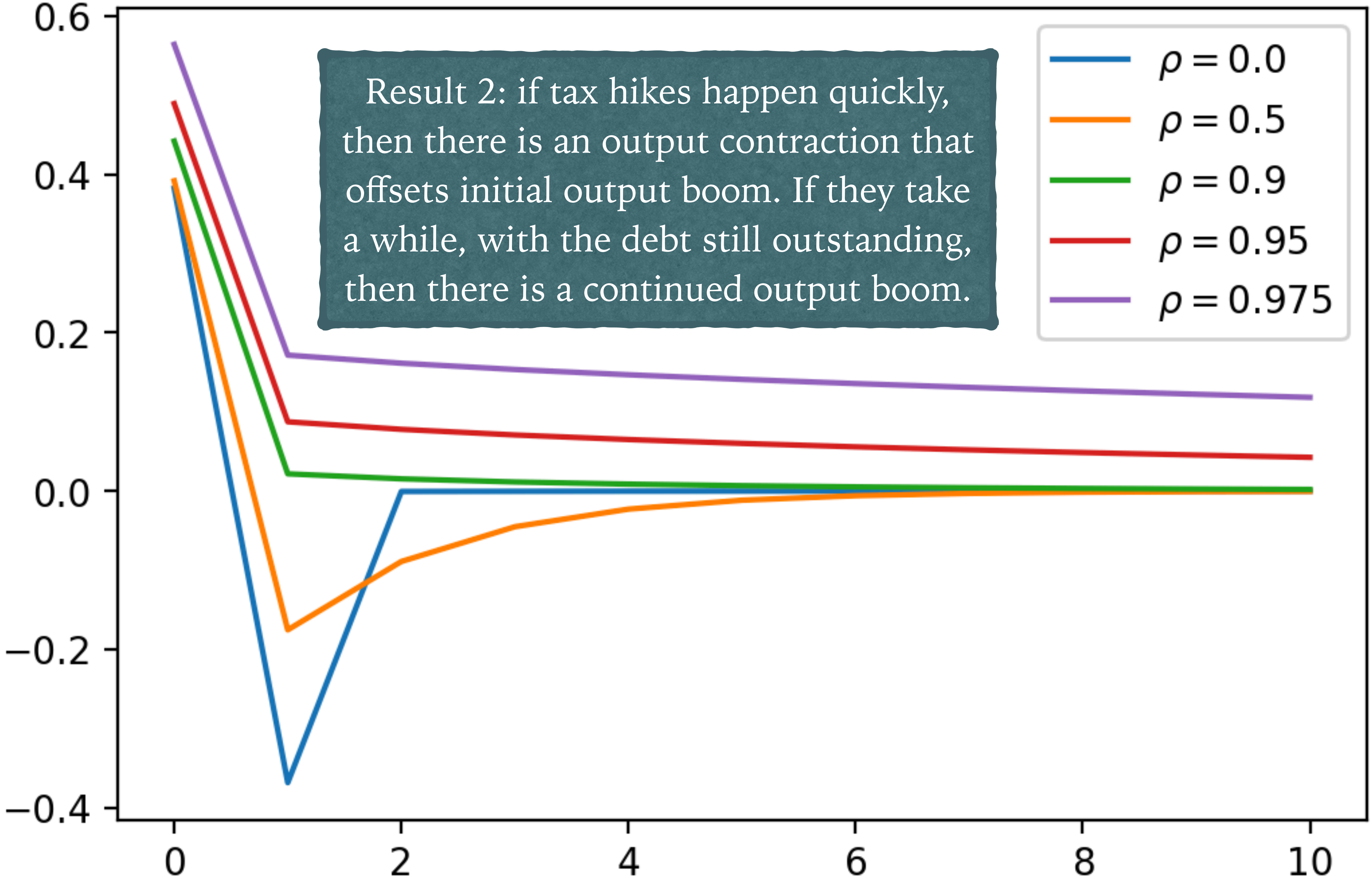


CLOSE CONNECTION BETWEEN SPENDING AND TAX MULTIPLIERS

- Recall that given spending $d\mathbf{G}$ and taxes $d\mathbf{T}$, there is some linear mapping that gives path of output

$$d\mathbf{Y} = \mathscr{M}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

- We know that if $d\mathbf{G} = d\mathbf{T}$, $d\mathbf{Y} = \mathscr{M}(d\mathbf{G} - \mathbf{M}d\mathbf{G}) = d\mathbf{G}$
- Can write

$$\begin{aligned} d\mathbf{Y} &= \mathscr{M}(d\mathbf{G} - \mathbf{M}d\mathbf{G} + \mathbf{M}(d\mathbf{G} - d\mathbf{T})) \\ &= d\mathbf{G} + \mathscr{M}\mathbf{M}(d\mathbf{G} - d\mathbf{T}) \end{aligned}$$

- So output always equals the "direct" effect of spending, plus something that depends on the *primary deficit* $d\mathbf{G} - d\mathbf{T}$

ALTERNATIVE MODELS: REPRESENTATIVE-AGENT AND TWO-AGENT

AN ALTERNATIVE MODEL: THE REPRESENTATIVE-AGENT (RA) MODEL

- Assuming that the real interest rate is held constant at its steady-state value, in a representative-agent model the household equalizes consumption across all periods.

$$dC_0 = dC_1 = \cdots \longrightarrow \sum_{t=0}^{\infty} (1+r)^{-t} dC_t = \frac{1+r}{r} dC_0$$

$$\sum_{t=0}^{\infty} (1+r)^{-t} dC_t = \sum_{t=0}^{\infty} (1+r)^{-t} (dY_t - dT_t)$$

$$dC_0 = dC_1 = \cdots = \frac{r}{1+r} \sum_{t=0}^{\infty} (1+r)^{-t} (dY_t - dT_t)$$

REPRESENTATIVE-AGENT MODEL CONTINUED...

$$dC_0 = dC_1 = \cdots = \frac{r}{1+r}\sum_{t=0}^{\infty}(1+r)^{-t}(dY_t - dT_t)$$

$$= (1-\beta)\sum_{t=0}^{\infty}\beta^t(dY_t - dT_t)$$

$$\mathbf{M} = (1-\beta)\begin{pmatrix} 1 & \beta & \beta^2 & \cdots \\ 1 & \beta & \beta^2 & \cdots \\ 1 & \beta & \beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \longleftrightarrow \begin{matrix} \mathbf{M}^{RA} = (1-\beta)\mathbf{1}\mathbf{q}' \\ \mathbf{q}' \equiv (1, \beta, \beta^2, \ldots) \end{matrix}$$

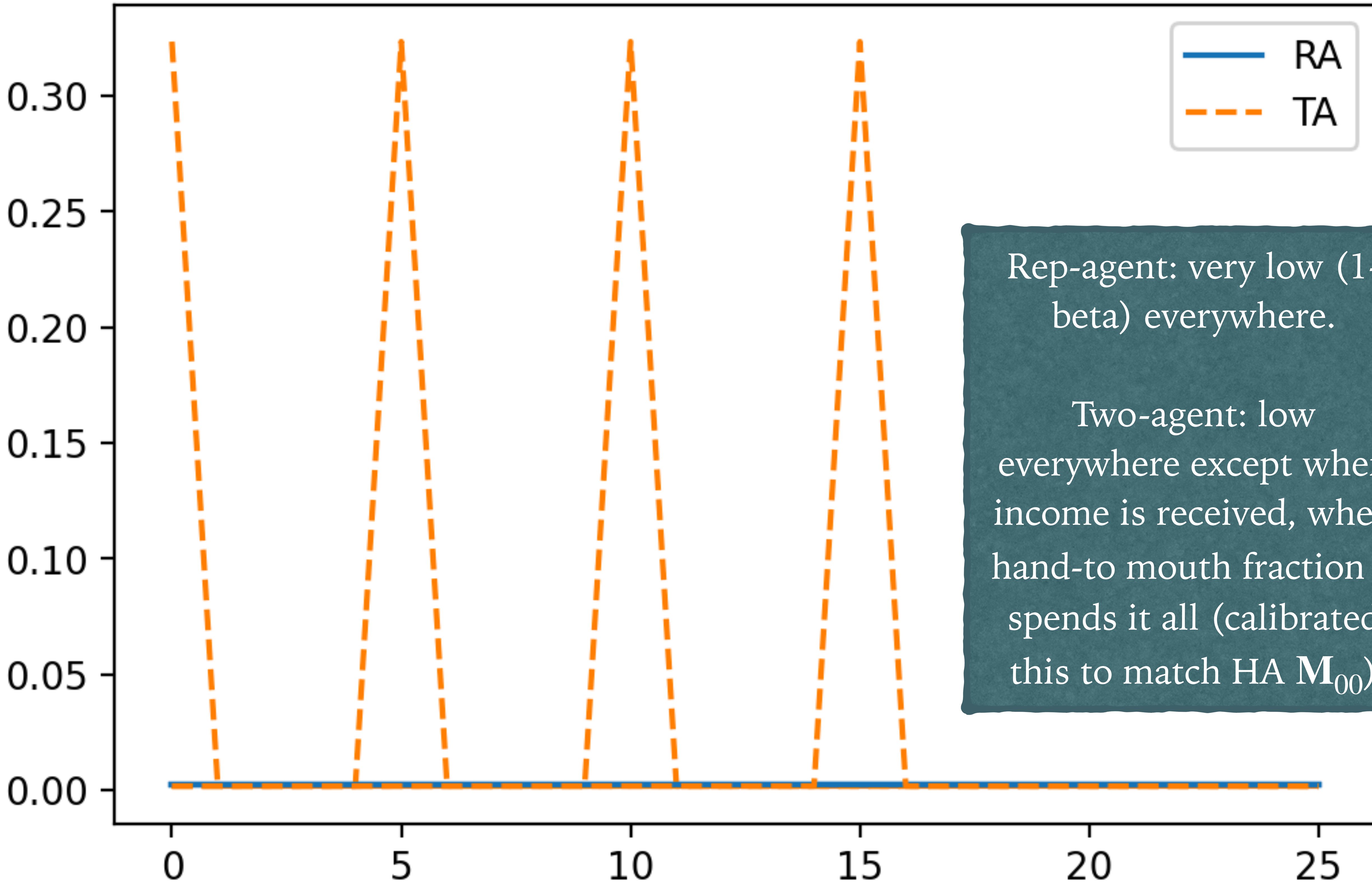
TWO-AGENT MODEL (TA)

- Now suppose that some measure μ of households are "hand-to-mouth", meaning that they consume exactly their income every period
  - these households have $\mathbf{M} = \mathbf{I}$, the identity matrix
  - assume remaining $1 - \mu$ is representative agent
  - sometimes called "spender-saver" model (Mankiw 2000, Gali, Lopez-Salido, Valles 2007)

- $\mathbf{M}$ matrix is given by

$$\mathbf{M}^{TA} = \mu \mathbf{I} + (1 - \mu)\mathbf{M}^{RA}$$

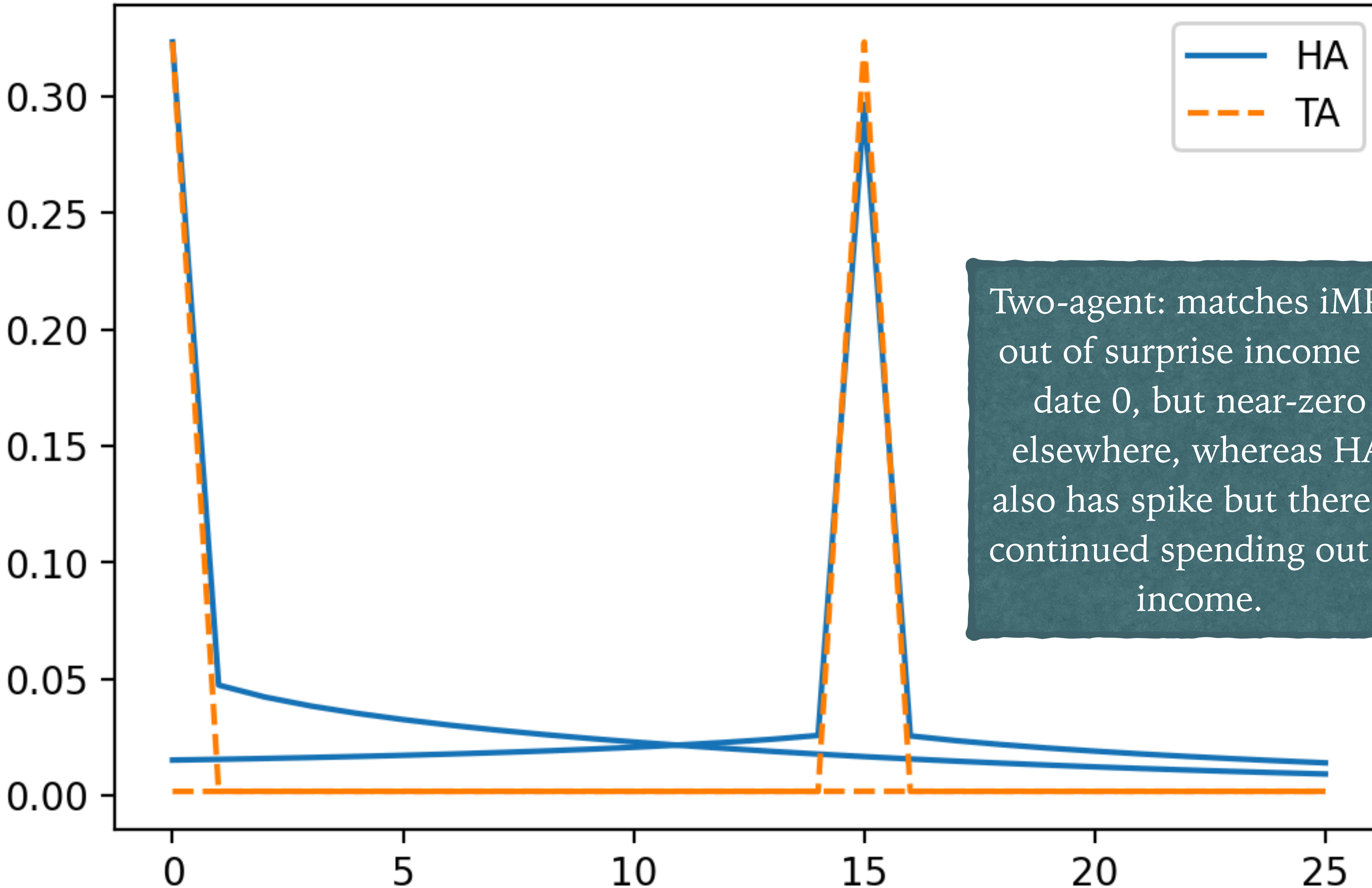
REP-AGENT (RA) VS. TWO-AGENT (TA) IMPCS



Rep-agent: very low (1-beta) everywhere.

Two-agent: low everywhere except when income is received, when hand-to mouth fraction μ spends it all (calibrated this to match HA $\mathbf{M}_{00}$)

WHAT ABOUT TWO-AGENT (TA) VS. HET-AGENT (HA)?



Two-agent: matches iMPC out of surprise income at date 0, but near-zero elsewhere, whereas HA also has spike but there is continued spending out of income.

WHAT IS THE MULTIPLIER IN A RA MODEL? ALWAYS 1 ON SPENDING

- Write $\mathbf{M}^{RA} = (1-\beta)\mathbf{1}\mathbf{q}'$ and plug into the IKC:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^{RA}d\mathbf{T} + \mathbf{M}^{RA}d\mathbf{Y} \quad \longleftrightarrow \quad d\mathbf{Y} = d\mathbf{G} + (1-\beta)\mathbf{1}\mathbf{q}'(d\mathbf{Y} - d\mathbf{T})$$

- Conjecturing that $d\mathbf{Y} = d\mathbf{G}$, reduces to just

$$\mathbf{q}'(d\mathbf{G} - d\mathbf{T}) = 0$$

which is gov present-value budget balance, which we require!

- Hence $d\mathbf{Y} = d\mathbf{G}$ is a solution! (also in Woodford 2011)
  - Taxes don't matter, same as balanced-budget
  - Because of Ricardian equivalence in RA model

WHAT ABOUT MULTIPLIER IN TA MODEL?

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^{TA}d\mathbf{T} + \mathbf{M}^{TA}d\mathbf{Y}$$

$$\longleftrightarrow \quad d\mathbf{Y} = d\mathbf{G} - (1-\mu)\mathbf{M}^{RA}d\mathbf{T} + (1-\mu)\mathbf{M}^{RA}d\mathbf{Y} - \mu d\mathbf{T} + \mu d\mathbf{Y}$$

$$\longleftrightarrow \quad (1-\mu)d\mathbf{Y} = d\mathbf{G} - \mu d\mathbf{T} - (1-\mu)\mathbf{M}^{RA}d\mathbf{T} + (1-\mu)\mathbf{M}^{RA}d\mathbf{Y}$$

$$\longleftrightarrow \quad d\mathbf{Y} = \left(\frac{d\mathbf{G} - \mu d\mathbf{T}}{1-\mu}\right) - \mathbf{M}^{RA}d\mathbf{T} + \mathbf{M}^{RA}d\mathbf{Y}$$

$$\longrightarrow \quad d\mathbf{Y} = \mathscr{M}^{RA}\left(\left(\frac{d\mathbf{G} - \mu d\mathbf{T}}{1-\mu}\right) - \mathbf{M}^{RA}d\mathbf{T}\right)$$

$$\longrightarrow \quad d\mathbf{Y} = \frac{d\mathbf{G} - \mu d\mathbf{T}}{1-\mu}$$

INTERPRETING THIS EXPRESSION IN TA MODEL

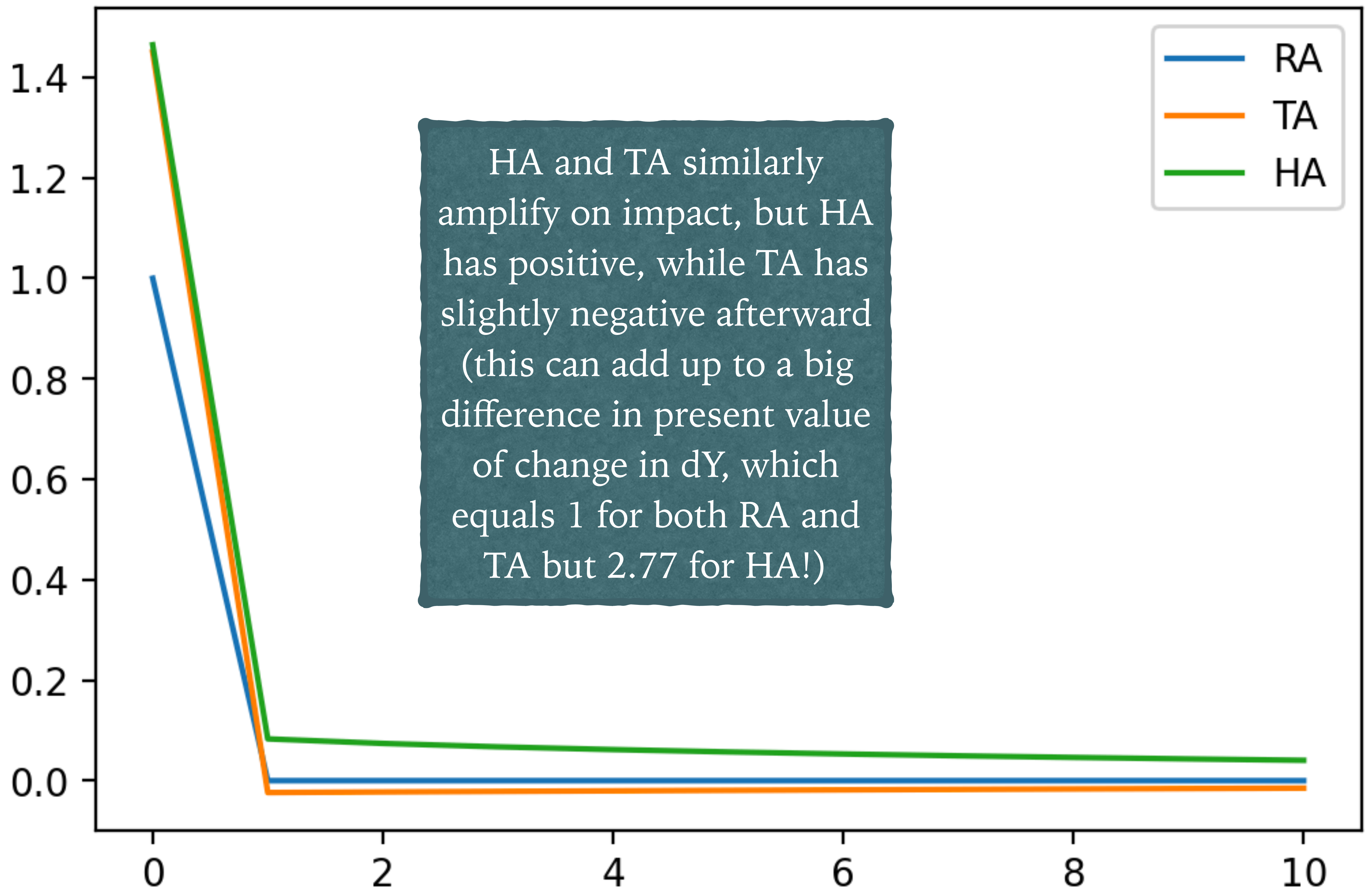
Exactly the same, period by period, as "static" traditional Keynesian cross, with μ replacing "mpc"

$$d\mathbf{Y} = \frac{d\mathbf{G} - \mu d\mathbf{T}}{1 - \mu} \qquad \longleftrightarrow \qquad dy = \frac{dg - mpc \cdot dt}{1 - mpc}$$

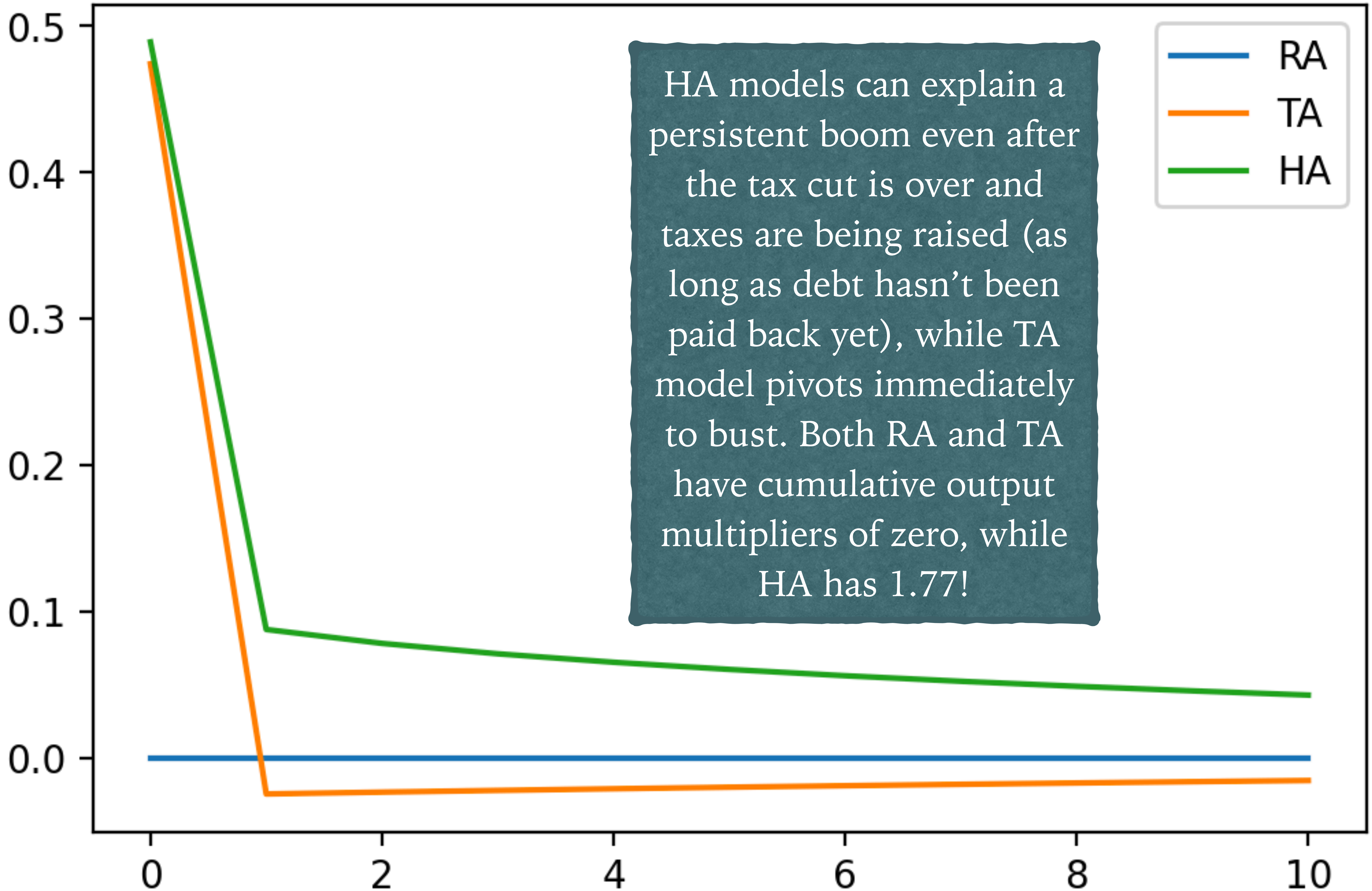
Multiplier on spending, if no taxes, is $1/(1 - \mu)$

Multiplier on taxes is $-\mu/(1 - \mu)$

RA VS TA VS HA MULTIPLIERS ON PERSISTENT DEFICIT-FINANCED G



SAME EXERCISE FOR A TAX CUT



HA models can explain a persistent boom even after the tax cut is over and taxes are being raised (as long as debt hasn't been paid back yet), while TA model pivots immediately to bust. Both RA and TA have cumulative output multipliers of zero, while HA has 1.77!

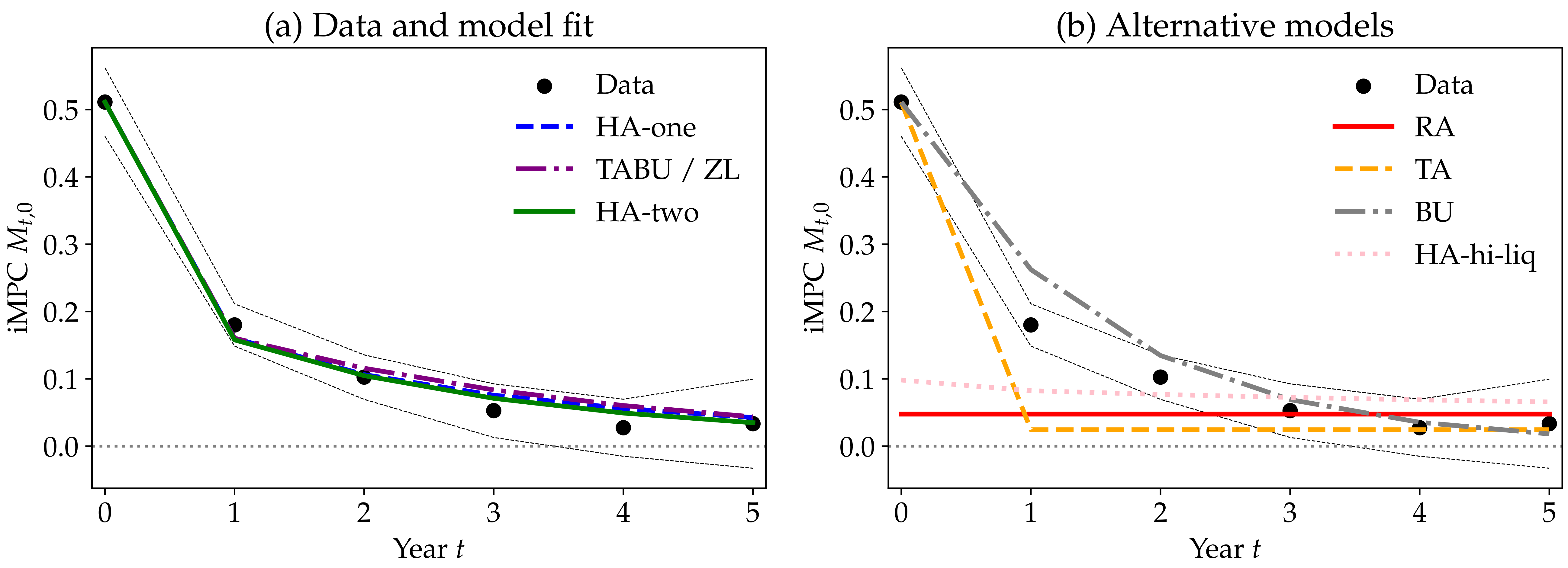
WHAT'S GOING ON?

- In TA models, just like in RA models, the **cumulative** multiplier on spending equals 1 and on taxes equals 0
- This is because these models also amplify the contractionary effects of later tax increases, which offsets the expansionary effects of tax cuts or spending

- In HA models, the **cumulative** multiplier is much higher
  - need to convince households with targets for assets to hold extra bonds, which requires a boom
  - **explains why huge deficit spending (2020-21) can have persistent effects on demand today!**

WHICH MODELS HAVE REALISTIC IMPCS? AUCLERT-ROGNLIE-STRAUB 2024

Figure 2: iMPCs in the Norwegian data and in several models



Several other models here, but we can see that an HA model (the one we're using in these slides is "HA-one") can mostly fit the empirical impulse response to an unexpected income shock, while RA and TA models do not.