

INTRODUCTION TO MONETARY POLICY IN HANK

Econ 411-3
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TURNING TO MONETARY POLICY: TRANSMISSION OF REAL RATES

- Last time we introduced a “canonical” HANK model that embedded the standard incomplete markets model
- Policy variables included:
 - **Fiscal:** $d\mathbf{B}$, $d\mathbf{G}$, $d\mathbf{T}$
 - **Monetary:** dr
- We focused on **fiscal** and showed big differences between HANK, TANK, and RANK
- Now we’ll talk about **monetary** policy in the same model, interpreted as the effect of changing the real rate (which here equals the nominal rate)

PREVIEW: A CHANGE IN MECHANISMS

- In the basic representative-agent NK model, monetary policy operated through the intertemporal substitution channel
 - if you knew the path of real interest rates, you'd know consumption via the Euler equation
- In heterogeneous-agent models, a number of other channels are now possible, and indeed often dominate. These include:
 - income effects from rates: real interest rates redistribute between heterogeneous agents
 - income effects from general equilibrium changes in income
 - income effects from changes in taxes

REFRESHER: DERIVING THE INTERTEMPORAL KEYNESIAN CROSS

- Last lecture, we reduced the economy to either a single sequence-space system for goods market clearing, or assets:

$$Y_t = \mathcal{C}_t(\{Y_s - T_s\}, \{r_s^{ante}\}) + G_t$$

$$B_t = \mathcal{A}_t(\{Y_s - T_s\}, \{r_s^{ante}\})$$

- Then we assumed real interest rates were constant and took a first-order approximation of the goods equation, stacking in vectors and defining \mathbf{M} by $M_{ts} \equiv \partial \mathcal{C}_t / \partial (Y_s - T_s)$:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- This is the “intertemporal Keynesian cross” (IKC)

NOW: SAME THING, BUT DON'T ASSUME R CONSTANT

- Now let's not assume constant r here:

$$Y_t = \mathcal{C}_t(\{Y_s - T_s\}, \{r_s^{ante}\}) + G_t$$

- Instead, define $M_{ts}^r \equiv \partial \mathcal{C}_t / \partial \log(1 + r_s^{ante})$
 - also stack $d\mathbf{r}$ with entries $[d\mathbf{r}]_s = dr_s^{ante} / (1 + r)$
 - then linearize to obtain generalized IKC:

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- Exactly the same, but now takes into account monetary policy
 - whose demand effects enter same as fiscal policy!

SOLUTION STILL THE SAME

- Generalized IKC:

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- Same solution for some \mathcal{M} :

$$d\mathbf{Y} = \mathcal{M}(\mathbf{M}^r d\mathbf{r} + d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

- Given a “partial equilibrium” demand effect $\mathbf{M}^r d\mathbf{r}$ of monetary policy, a fiscal shock with the same PE effect $d\mathbf{G} - \mathbf{M}d\mathbf{T}$ will also have same GE outcome

- Wolf (AER 2023) dubs this property “demand equivalence”

IMPORTANT FISCAL COMPLICATION

- Important complication in period-by-period government budget constraint, which to first order is now:

$$B_t - (1 + r_{t-1}^{ante})B_{t-1} = G_t - T_t$$

$$dB_t - (1 + r)dB_{t-1} - dr_{t-1}^{ante} B = dG_t - dT_t$$

- ... so: unless $B = 0$, monetary shock **requires** fiscal response, with changing $dG_t - dT_t$ at some point to offset changing r_t^{ante}

- Assume fiscal authority aims for balanced budget coming into period, setting sum of two last terms on left to zero:

$$dB_t = -\frac{1}{1+r} dr_t^{ante} B \quad \longleftrightarrow \quad d\mathbf{B} = -Bd\mathbf{r}$$

FISCAL ASSUMPTIONS CONTINUED

- Recall government budget constraint

$$dB_t - (1 + r)dB_{t-1} - dr_{t-1}^{ante}B = dG_t - dT_t$$

- We assume

$$dB_t = -\frac{1}{1+r}dr_t^{ante}B \quad \longleftrightarrow \quad d\mathbf{B} = -Bd\mathbf{r}$$

- This reduces constraint above to just

$$dB_t = dG_t - dT_t \quad \longleftrightarrow \quad d\mathbf{B} = d\mathbf{G} - d\mathbf{T}$$

- Government in principle can adjust either $d\mathbf{G}$ or $d\mathbf{T}$, i.e. either spending or taxes

- we'll mostly assume taxes, but can consider spending too

ANALYTICAL MODELS: RA AND TA

REPRESENTATIVE-AGENT CASE

- We already know representative agent obeys Euler equation

$$C_t^{-\sigma} = \beta(1 + r_t^{ante})C_{t+1}^{-\sigma}$$
$$dC_t = -\sigma^{-1}C \frac{dr_t^{ante}}{1+r} + dC_{t+1}$$

- This tells us what happens to consumption in GE, assuming return to steady state for high t
- Can write this in matrix notation if desired as

$$d\mathbf{C} = -\sigma^{-1}\mathbf{C}\mathbf{U}d\mathbf{r}$$

where \mathbf{U} is matrix with 1s on and above the diagonal

REPRESENTATIVE-AGENT CASE, CONTINUED

- Equilibrium output is just the sum of this and spending:

$$d\mathbf{Y} = d\mathbf{C} + d\mathbf{G} = -\sigma^{-1}C\mathbf{U}d\mathbf{r} + d\mathbf{G}$$

- Can compare to generalized IKC:

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- given our formula for the RA \mathbf{M} from last time, easy to calculate $\mathbf{M}d\mathbf{T}$ and $\mathbf{M}d\mathbf{Y}$, can infer $\mathbf{M}^r d\mathbf{r}$ from that (or calculate it directly)

REPRESENTATIVE-AGENT CASE, CONTINUED

- Equilibrium output is just the sum of this and spending:

$$dY = dC + dG = -\sigma^{-1}CUdr + dG$$

- Can compare to generalized IKC:

$$dY = M^r dr + dG - MdT + MdY$$

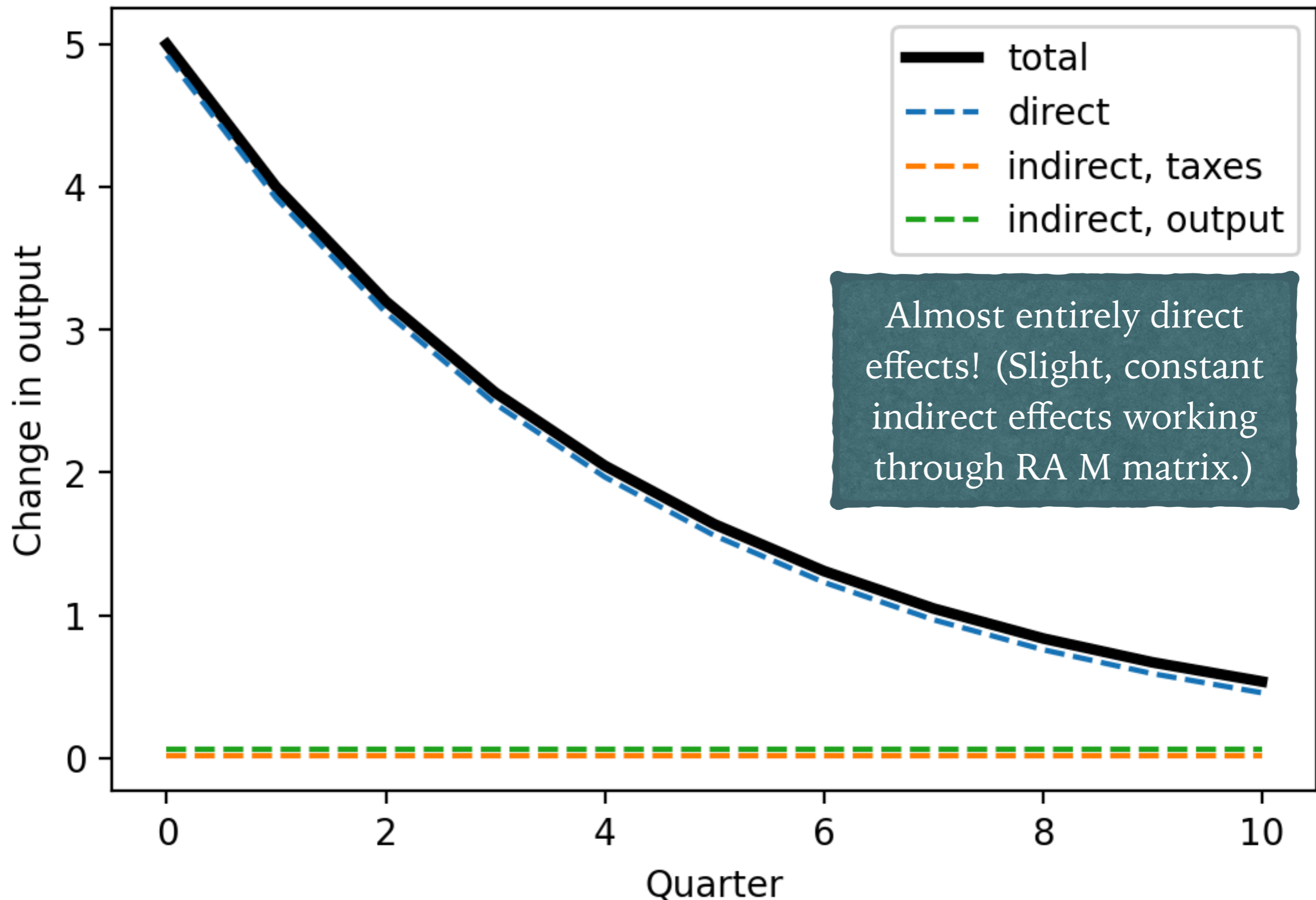
“Direct” effect of real interest rates (i.e. what shows up directly in household problem) on consumption

“Indirect” effects of real interest rates on consumption, working through changes in taxes or endogenous changes in output

(“Direct” vs. “indirect” nomenclature from Kaplan, Moll, Violante 2018)

EFFECT IN OUR CALIBRATION FROM LAST LECTURE

RA model, change in output from $dr_t = -0.8^t$



TWO-AGENT MODEL

- Let's now use the same two-agent model as last lecture
- Same strategy: write generalized IKC for TA, use fact that \mathbf{M} is identity for hand-to-mouth, and $\mathbf{M}^r = 0$; ignore G

$$d\mathbf{Y} = \mathbf{M}^{r,TA} d\mathbf{r} + d\mathbf{G} - \mathbf{M}^{TA} d\mathbf{T} + \mathbf{M}^{TA} d\mathbf{Y}$$

$$d\mathbf{Y} = (1 - \mu)\mathbf{M}^{r,RA} d\mathbf{r} - (1 - \mu)\mathbf{M}^{RA} d\mathbf{T} + (1 - \mu)\mathbf{M}^{RA} d\mathbf{Y} - \mu d\mathbf{T} + \mu d\mathbf{Y}$$

$$(1 - \mu)d\mathbf{Y} = (1 - \mu)\mathbf{M}^{r,RA} d\mathbf{r} - (1 - \mu)\mathbf{M}^{RA} d\mathbf{T} + (1 - \mu)\mathbf{M}^{RA} d\mathbf{Y} - \mu d\mathbf{T}$$

$$d\mathbf{Y} = -\frac{\mu}{1 - \mu} d\mathbf{T} + \mathbf{M}^{r,RA} d\mathbf{r} - \mathbf{M}^{RA} d\mathbf{T} + \mathbf{M}^{RA} d\mathbf{Y}$$

Same as RA IKC with additional term from interaction of hand-to-mouth with taxes, so same effect as in RA with this additional term passed through with multiplier of 1 (like gov spending)

TWO-AGENT MODEL

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$$d\mathbf{Y} = \mathbf{M}^{r,TA} d\mathbf{r} + d\mathbf{G} - \mathbf{M}^{TA} d\mathbf{T} + \mathbf{M}^{TA} d\mathbf{Y}$$

$$d\mathbf{Y} = (1 - \mu)\mathbf{M}^{r,RA} d\mathbf{r} - (1 - \mu)\mathbf{M}^{RA} d\mathbf{T} + (1 - \mu)\mathbf{M}^{RA} d\mathbf{Y} - \mu d\mathbf{T} + \mu d\mathbf{Y}$$

$$(1 - \mu)d\mathbf{Y} = (1 - \mu)\mathbf{M}^{r,RA} d\mathbf{r} - (1 - \mu)\mathbf{M}^{RA} d\mathbf{T} + (1 - \mu)\mathbf{M}^{RA} d\mathbf{Y} - \mu d\mathbf{T}$$

$$d\mathbf{Y} = -\frac{\mu}{1 - \mu} d\mathbf{T} + \mathbf{M}^{r,RA} d\mathbf{r} - \mathbf{M}^{RA} d\mathbf{T} + \mathbf{M}^{RA} d\mathbf{Y}$$

$$d\mathbf{Y} = d\mathbf{Y}^{RA} - \frac{\mu}{1 - \mu} d\mathbf{T}$$

TWO-AGENT MODEL: CONCLUSION

- Same as representative-agent model except for tax changes in response to interest rates, which in our case we assume are

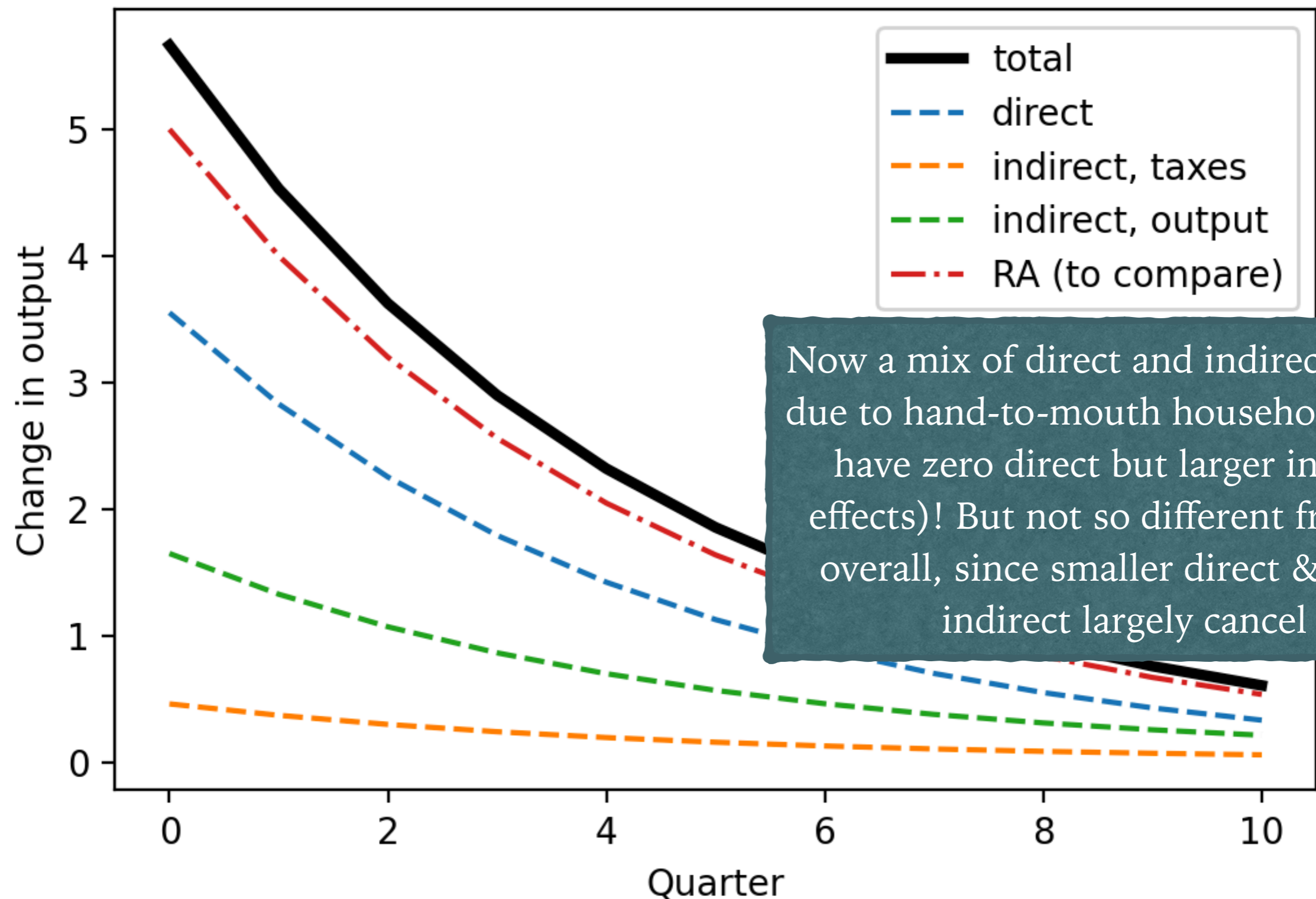
$$d\mathbf{Y} = d\mathbf{Y}^{RA} - \frac{\mu}{1 - \mu} d\mathbf{T} = d\mathbf{Y}^{RA} - \frac{\mu}{1 - \mu} B d\mathbf{r}$$

- So: the expansionary effect of a rate cut will be amplified by the effect of cutting taxes on hand-to-mouth households in response to lower interest expenditure
 - (This is assuming that all interest is earned by the non-hand-to-mouth household; otherwise, no effect.)
- On next slide we'll do same direct-indirect decomposition

$$d\mathbf{Y} = \mathbf{M}^{r,TA} d\mathbf{r} + d\mathbf{G} - \mathbf{M}^{TA} d\mathbf{T} + \mathbf{M}^{TA} d\mathbf{Y}$$

EFFECT IN TA MODEL (SAME CALIBRATION AS LAST LECTURE)

TA model, change in output from $dr_t = -0.8^t$



Now a mix of direct and indirect effects, due to hand-to-mouth households (who have zero direct but larger indirect effects)! But not so different from RA overall, since smaller direct & larger indirect largely cancel

SPECIAL CASE WITH ZERO DEBT IN EQUILIBRIUM

- In this case, no change in taxes to pay for debt, so

$$d\mathbf{Y}^{TA} = d\mathbf{Y}^{RA} - \frac{\mu}{1 - \mu} d\mathbf{T} = d\mathbf{Y}^{RA}$$

- Why this equivalence? Simple intuition!
 - Hand to mouth households only matter if they behave differently from representative agent
 - But in general equilibrium, if assets are zero, both hand-to-mouth and representative agent hold zero assets
 - So it can't matter if we replace some measure of rep agent with hand-to-mouth; must have same output effect!
 - (can't replace *all* households or else indeterminacy)

HET-AGENT (HA) MODEL

HA MODEL: MUST NOW PROCEED NUMERICALLY

- Still have same generalized IKC

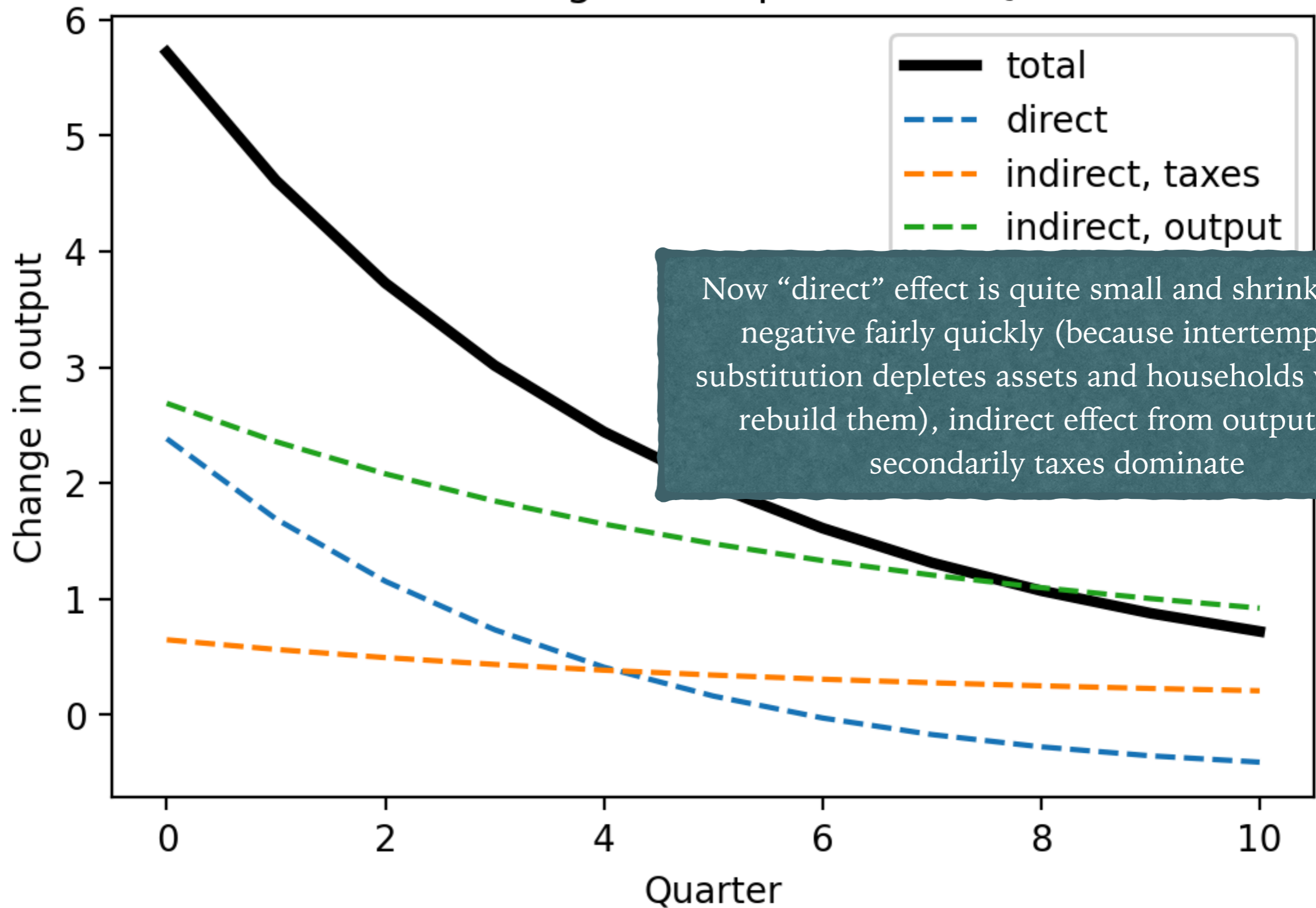
$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

but different matrices

- Can use same techniques to solve as in fiscal case, just with additional shock to real interest rates
- Will do same decomposition again for the same shock

EFFECT IN HA MODEL (CALIBRATION SAME AS LAST LECTURE)

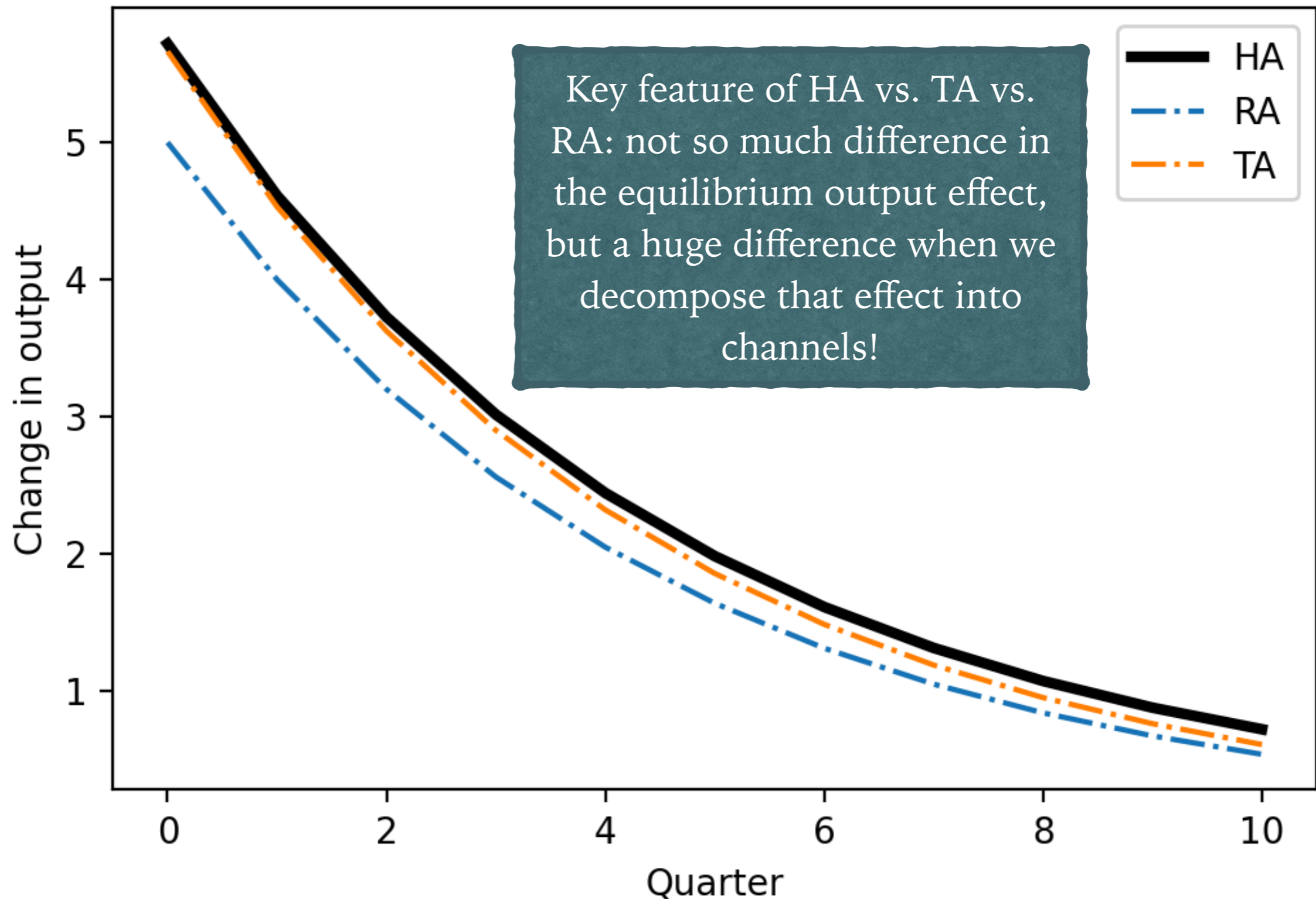
HA model, change in output from $dr_t = -0.8^t$



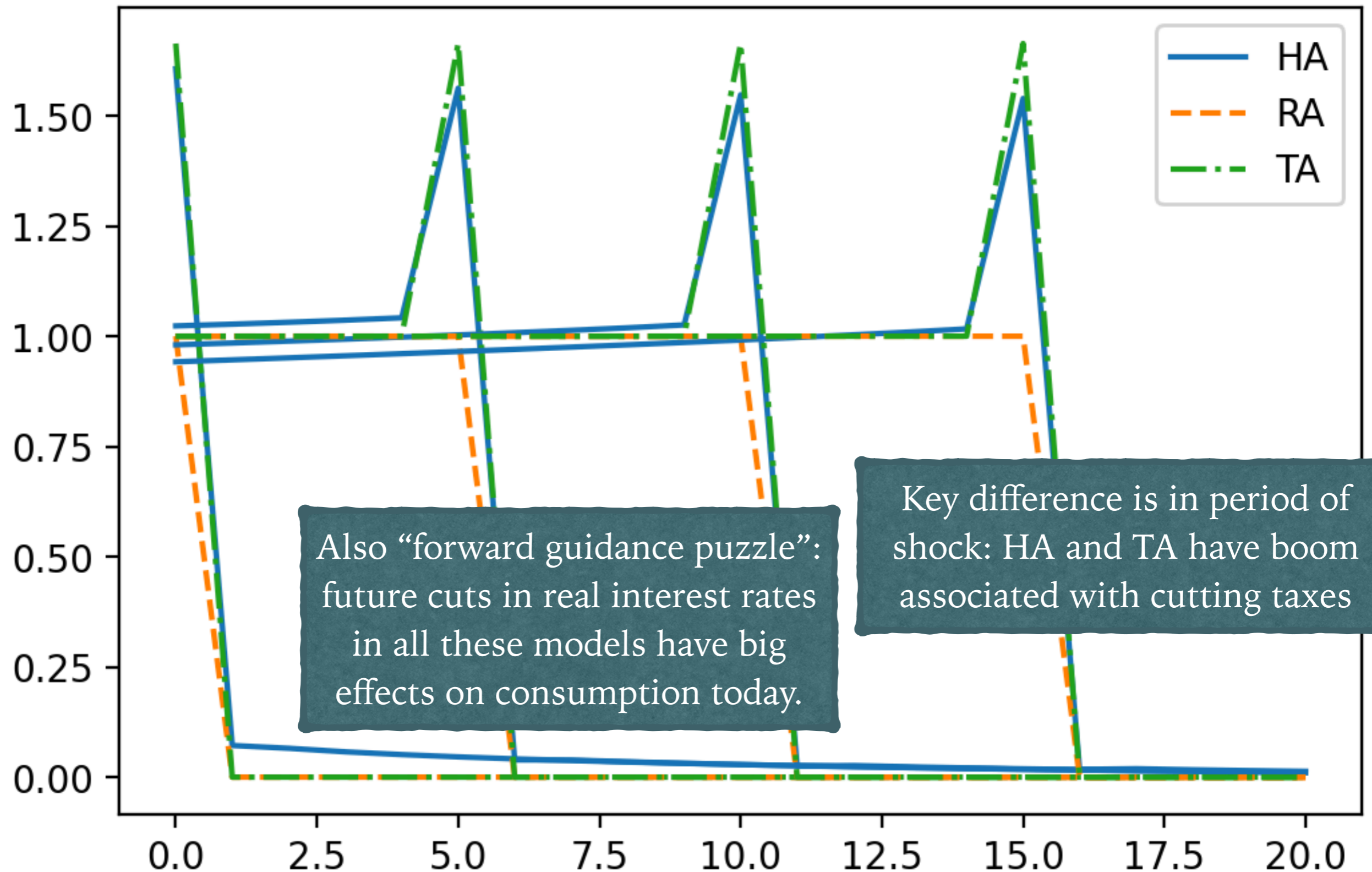
Now “direct” effect is quite small and shrinks to be negative fairly quickly (because intertemporal substitution depletes assets and households want to rebuild them), indirect effect from output and secondarily taxes dominate

DIRECT EFFECT SMALLER, BUT EFFECT SIMILAR (BIGGER)

All models, change in output from $dr_t = -0.8^t$



WHAT IF WE LOOK AT INDIVIDUAL SHOCKS TO FUTURE R?



CHANGING FISCAL RULE

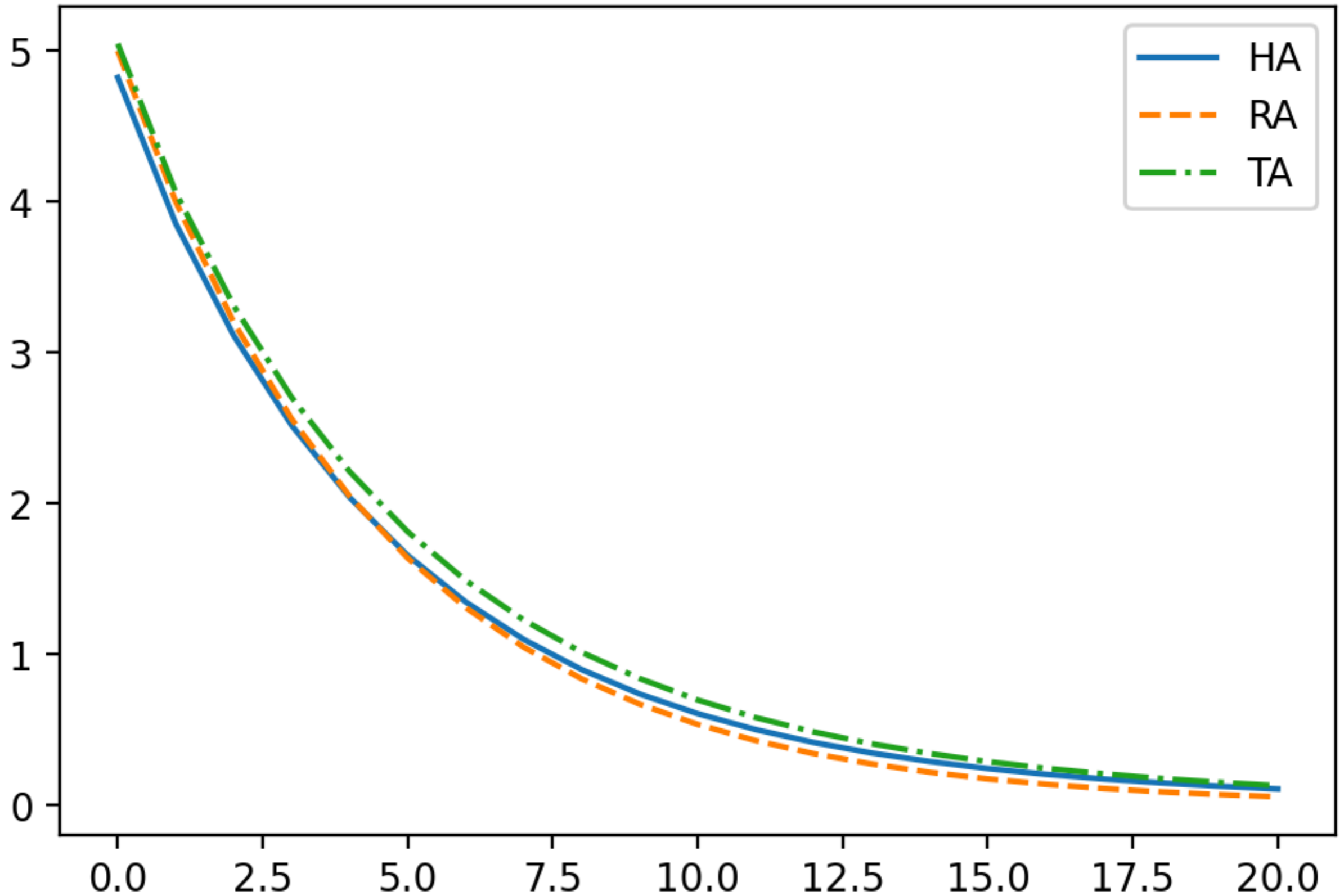
CONSIDER MORE GENERAL FISCAL RULE

- Write for some $\rho \in (0,1)$

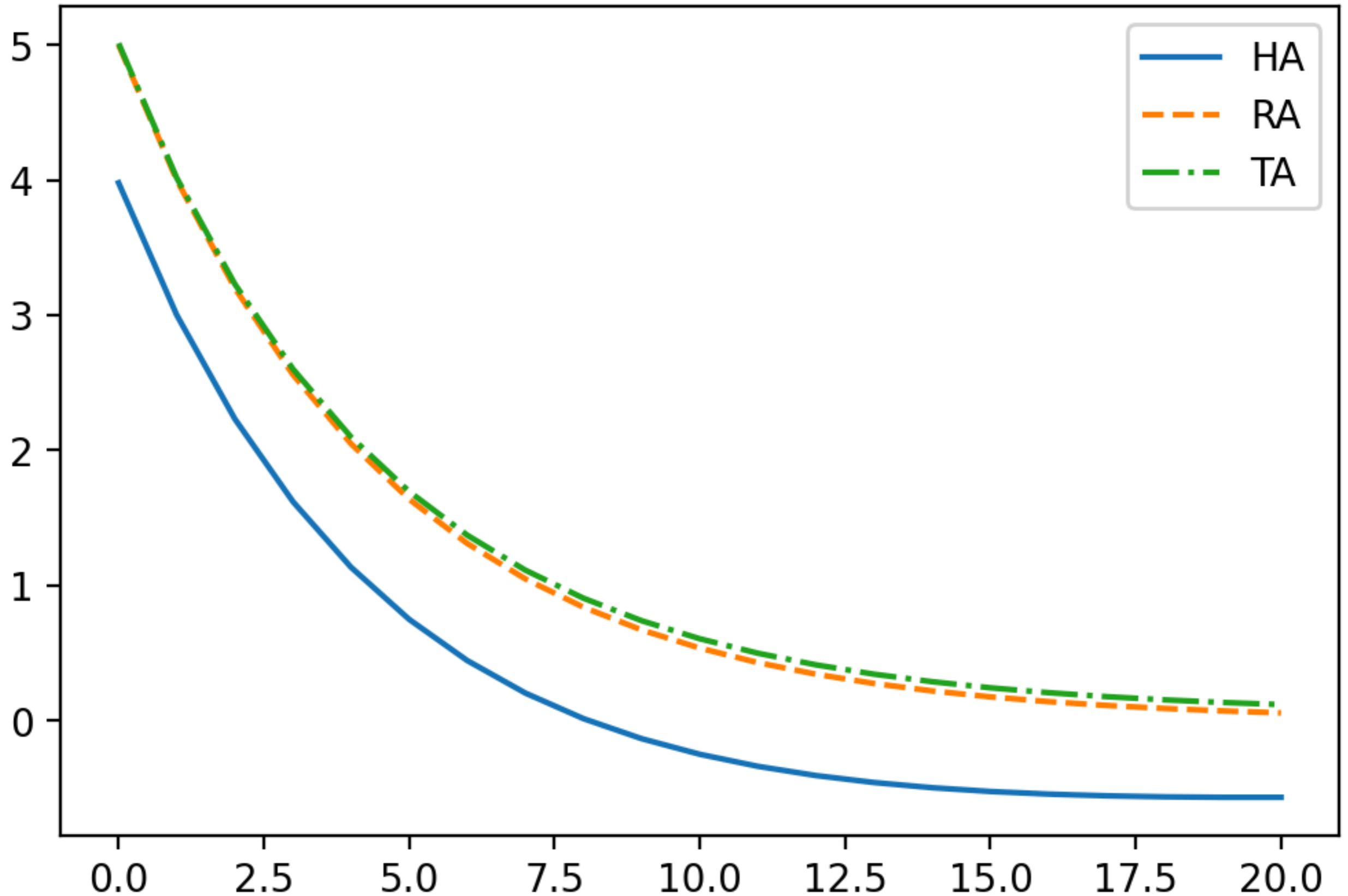
$$dB_t = \rho(dB_{t-1} + dr_{t-1}B)$$

- Here, with $\rho > 0$, the government doesn't immediately change taxes to offset changes in interest rates
- Instead, debt adjusts to absorb the interest, and then gradually goes back to steady state—so the increased or decreased cost of interest shows up in taxes only with a delay
- Relative to before, this means that following an interest rate cut, it'll take longer for taxes to be cut, and debt will fall

WITH $\rho = 0.9$ (DEBT LASTS FOR 10 QUARTERS), APPROX EQUAL

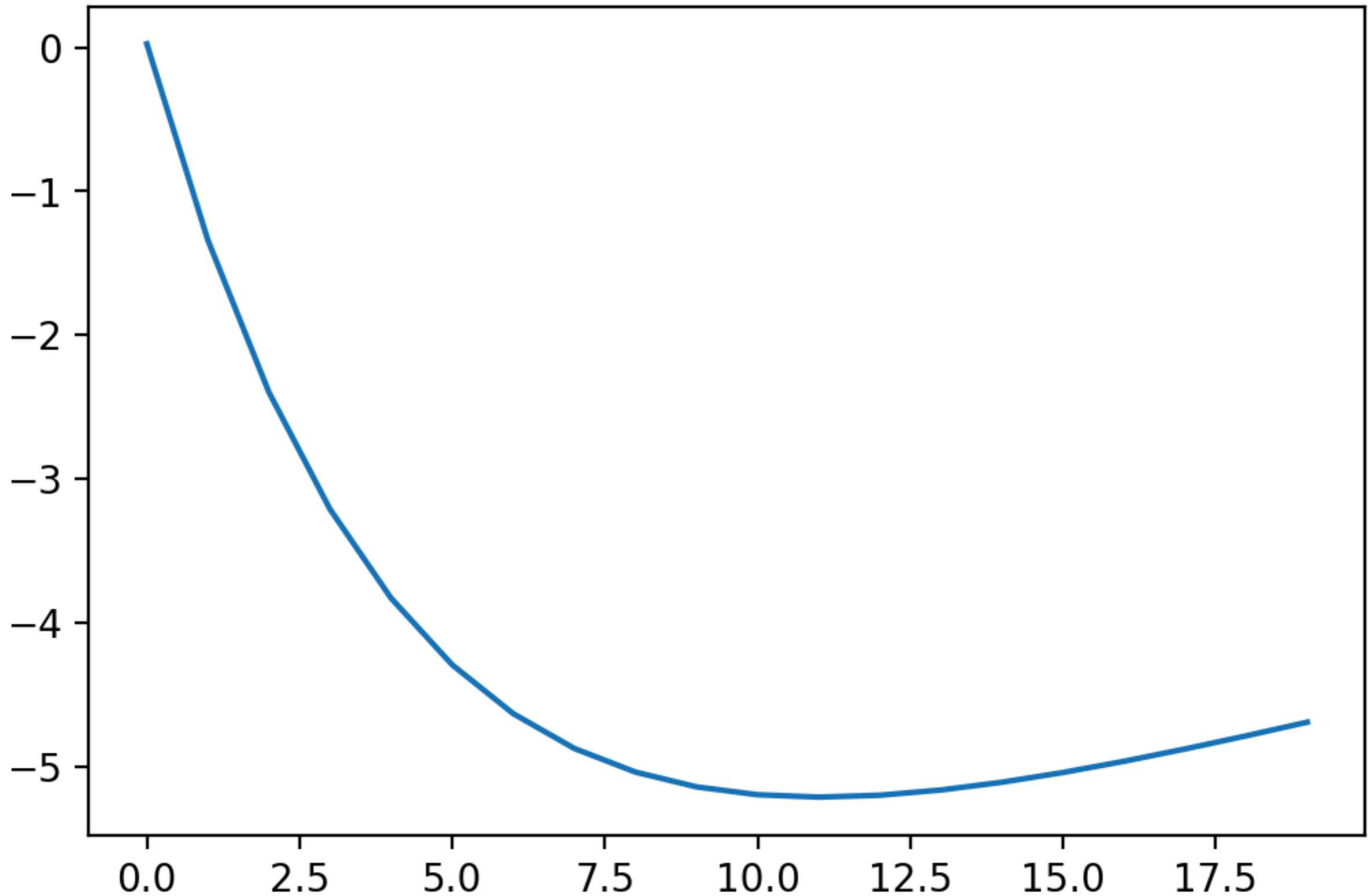


WITH $\rho = 0.975$ (DEBT LASTS FOR 40 QUARTERS), HA LOWER!

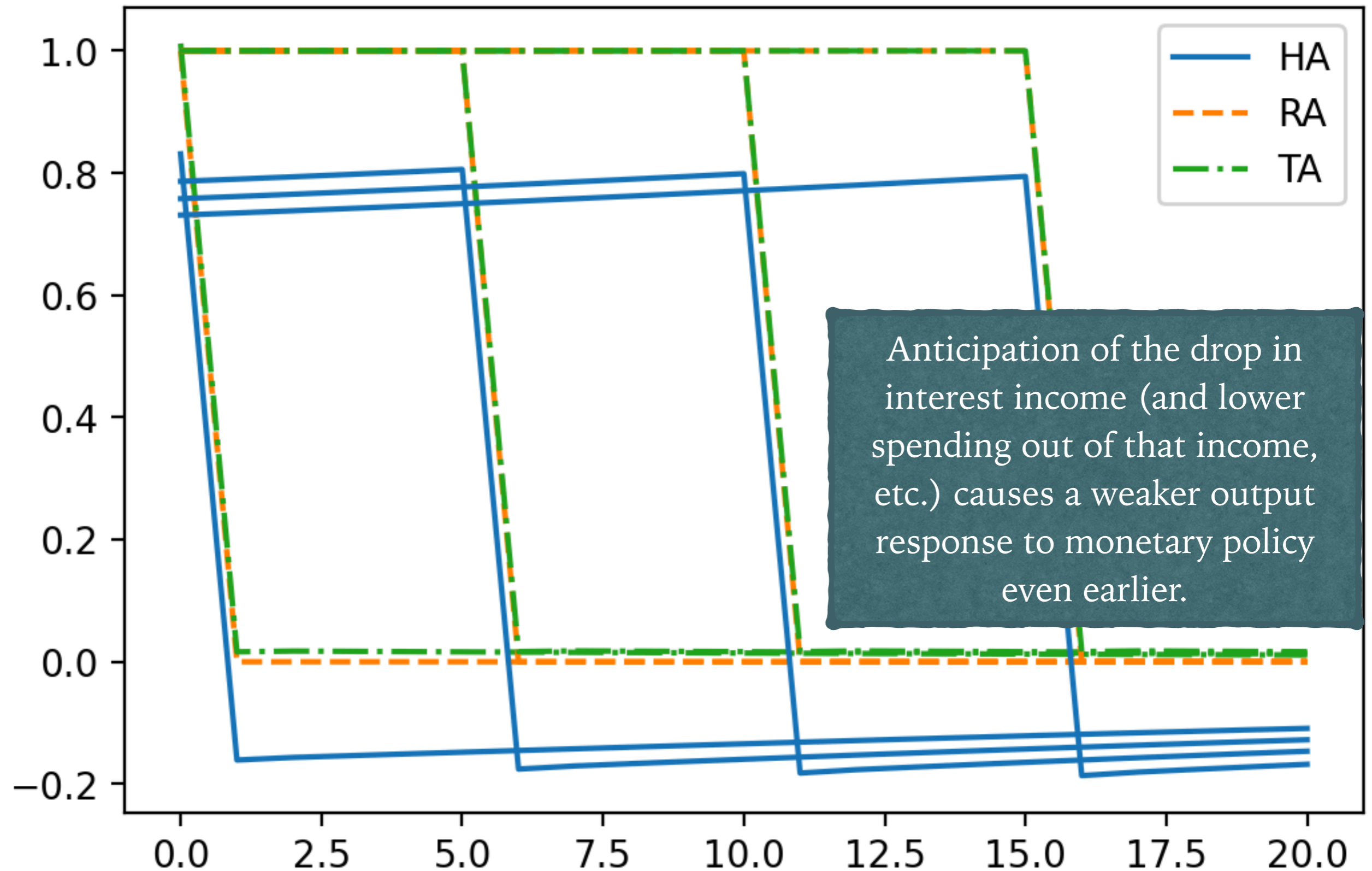


REASON: A PROLONGED DROP IN DEBT, WHICH IS CONTRACTIONARY IN HA

Path of debt for $\rho = 0.975$



CAN SEE IN IMPULSE RESPONSES TO INDIVIDUAL RATE SHOCKS



TAKEAWAY

- This is arguably not realistic for a short-lived monetary shock, because the government actually has locked in interest rates for several years with longer-term debt
 - Long-term debt is more complicated to model!
- Still, interesting: if fiscal policy delays the bounty of lower taxes in response to lower rates for long enough, dramatically lessens the output effect in HA model
 - Relative to baseline rule, it's a contractionary fiscal shock
- Note: this all works through “indirect” effects, not “direct” effects of interest rates!

A FEW MORE DECOMPOSITIONS

SUBSTITUTION VS. INCOME EFFECTS

- The “direct” effect of interest rates on households actually includes two conceptually separate effects
 - *substitution effects* working through the Euler equation
 - *income effects* working through the budget constraint
- Can separate these effects by pretending that a separate interest rate enters into each, then shocking each separately:

“Substitution” rate

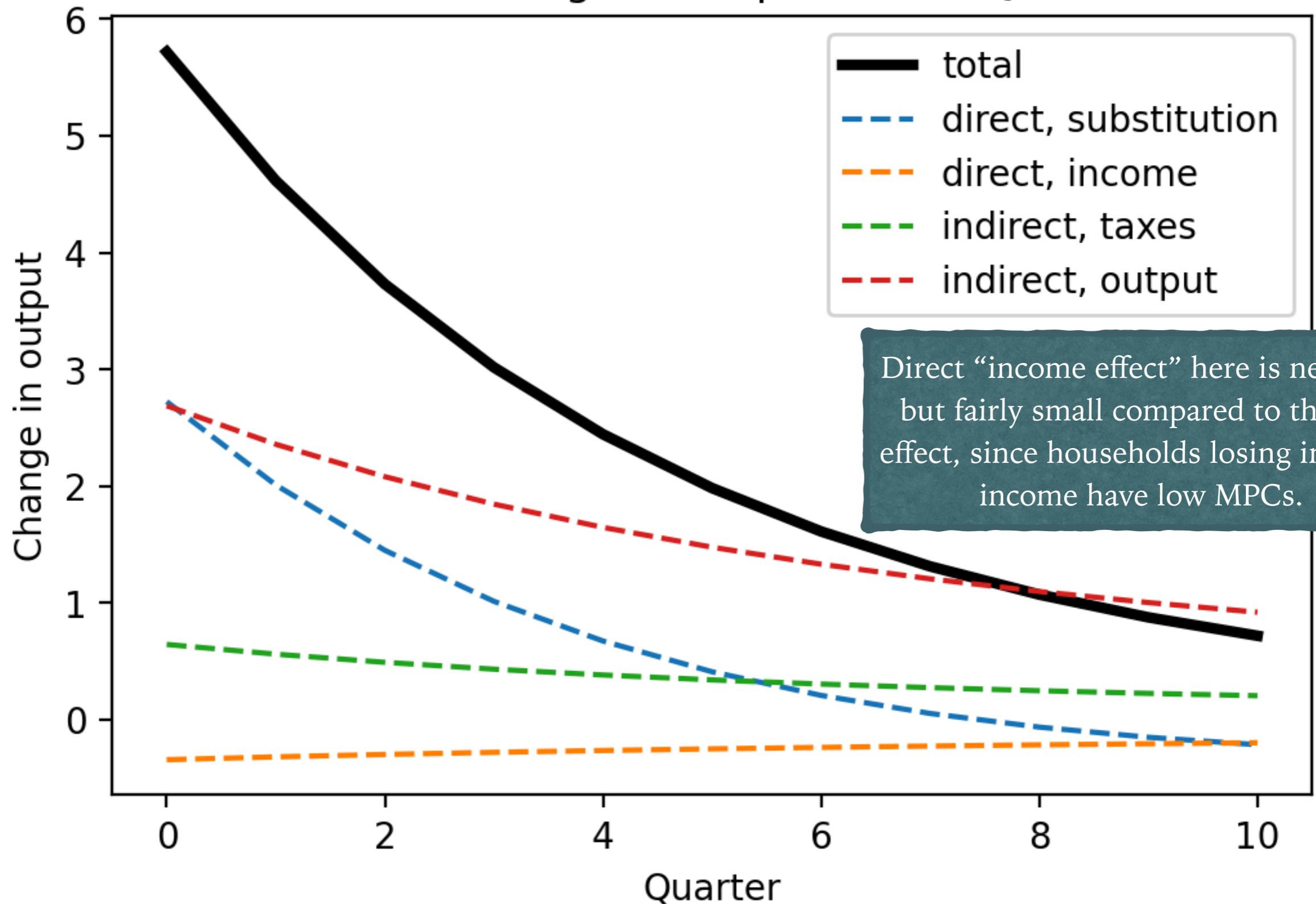
$$u'(c_{it}) \geq \beta(1 + r_{t-1}^{ante,sub})\mathbb{E}_t[u'(c_{i,t+1})]$$

“Income” rate

$$a_{it} + c_{it} = (1 + r_{t-1}^{ante,inc})a_{i,t-1} + y(s_{it})$$

IMPLEMENT DECOMPOSITION FOR ORIGINAL SHOCK, HA MODEL

HA model, change in output from $dr_t = -0.8^t$



Direct “income effect” here is negative but fairly small compared to the tax effect, since households losing interest income have low MPCs.

DECOMPOSING INDIRECT OUTPUT EFFECT

- We can view the previous decomposition as

$$d\mathbf{Y} = \mathbf{M}^{r,sub} d\mathbf{r} + \mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

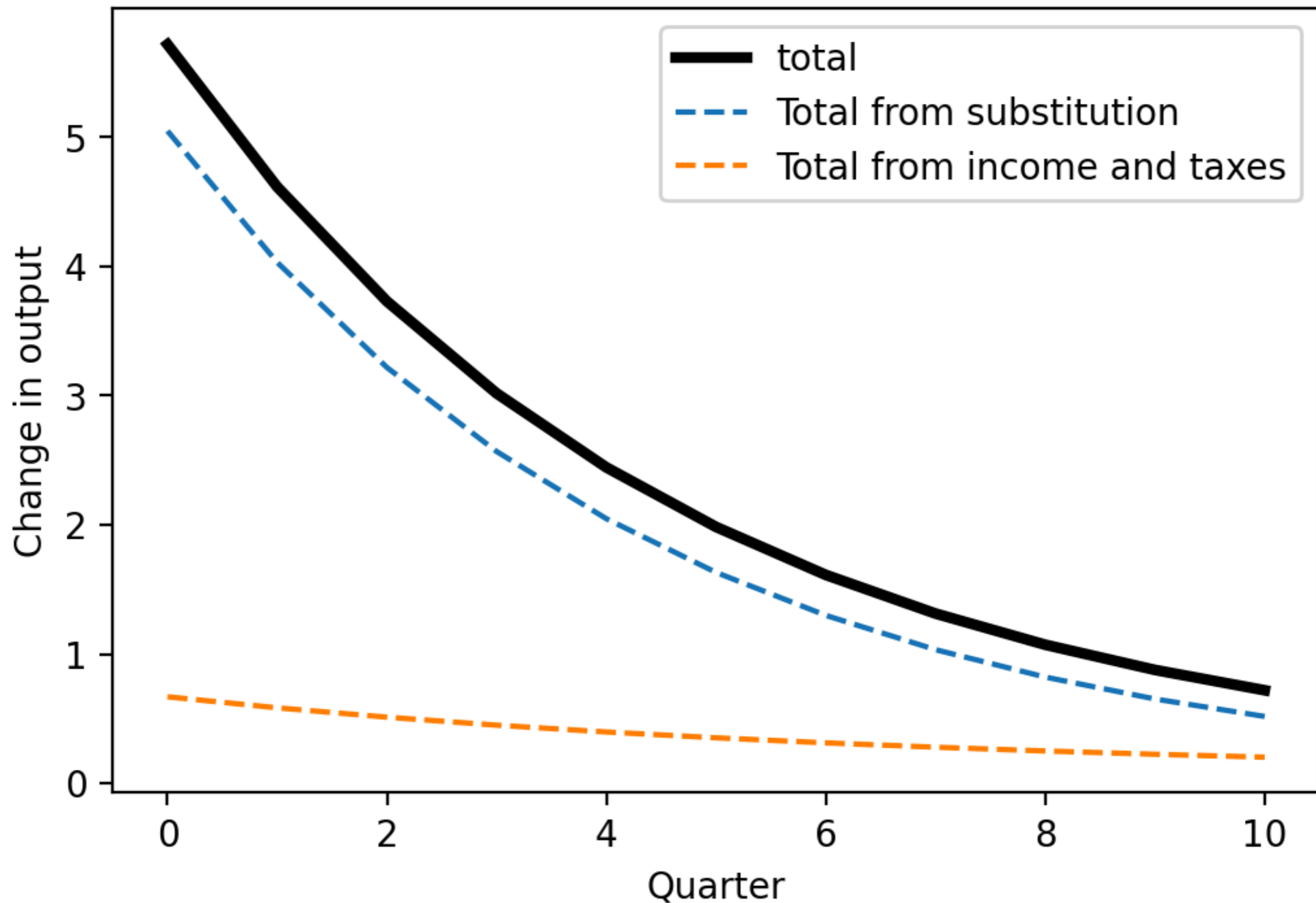
- But the “indirect effect” $d\mathbf{Y}$ is endogenous to everything else!
- Alternative: try to solve for GE separately from diff shocks:

$$d\mathbf{Y} = \underbrace{\mathcal{M}\mathbf{M}^{r,sub} d\mathbf{r}}_{\text{GE effect of substitution}} + \underbrace{\mathcal{M}(\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T})}_{\text{GE effect of income \& taxes}}$$

- Why do we need to bundle income effects and taxes together?
 - can only solve IKC if shock has net present value zero
 - conceptually: need a well-defined shock doesn't conjure income out of thin air

IMPLEMENT FOR ORIGINAL SHOCK, HA MODEL

HA model, change in output from $dr_t = -0.8^t$



CAN USE TO GENERALIZE OUR ORIGINAL DECOMPOSITION

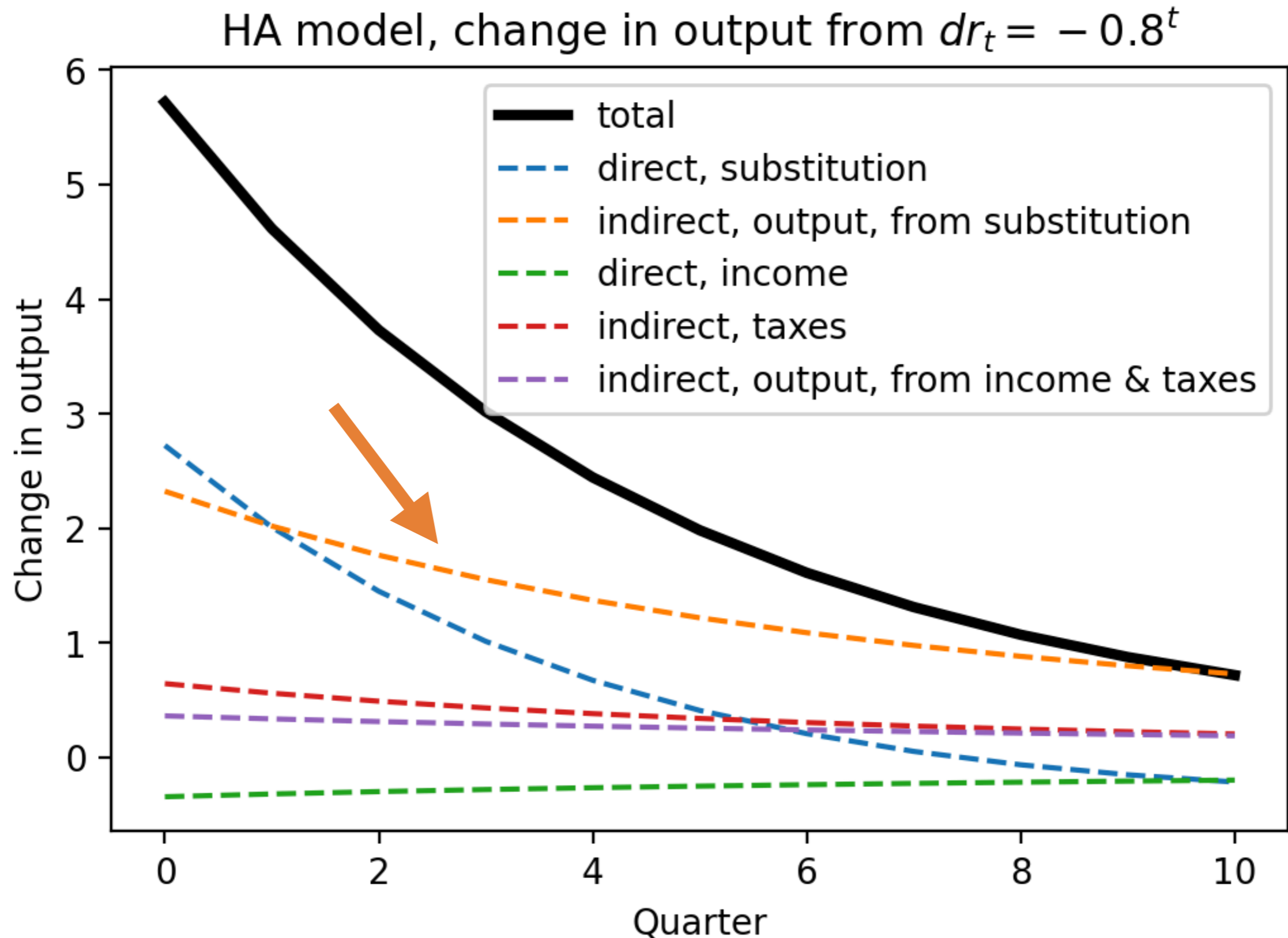
Direct substitution, income, and “indirect” tax effects of rates

$$\begin{aligned} dY = & \mathbf{M}^{r,sub} d\mathbf{r} + \mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T} \\ & + (\mathcal{M}\mathbf{M}^{r,sub} d\mathbf{r} - \mathbf{M}^{r,sub} d\mathbf{r}) \\ & + (\mathcal{M}(\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T}) - (\mathbf{M}^{r,inc} d\mathbf{r} - \mathbf{M}d\mathbf{T})) \end{aligned}$$

*Indirect effect
from output
ultimately
attributable to
substitution*

*Indirect effect from output
ultimately attributable to
income & taxes*

REVEALS: BIGGEST ROLE FOR INDIRECT EFFECTS TRIGGERED BY SUBSTITUTION



WHAT HAVE WE LEARNED?

- In our main HA calibration:
 - most effect still comes either directly from substitution, or indirectly from changes in output ultimately triggered by substitution
 - combined effects of income & taxes relatively small, but go in same direction as substitution
 - if government cuts taxes with a delay, this latter effect can be reversed

TWO THINGS TO CONSIDER

- Perhaps “income effects” are small because we’re missing some important channels (Auclert 2019)
 - we don’t have private debt; if we did, debtors might have higher MPCs, so cutting rates would have expansionary effect, especially with adjustable-rate mortgages, etc.
 - inflation might erode real value of debt, creating more space for borrowing
- Strong indirect effects triggered by substitution rely heavily on rational expectations (e.g. Farhi and Werning 2019)
 - if households don’t know output will expand so much, they won’t spend as much in anticipation (and there will be a smaller, and delayed, output effect!)

OUTCOMES WITH MYOPIA

SIMPLE MODEL OF MYOPIA

- Assume households don't realize changes in income are coming, so that they always treat any income shock like a surprise

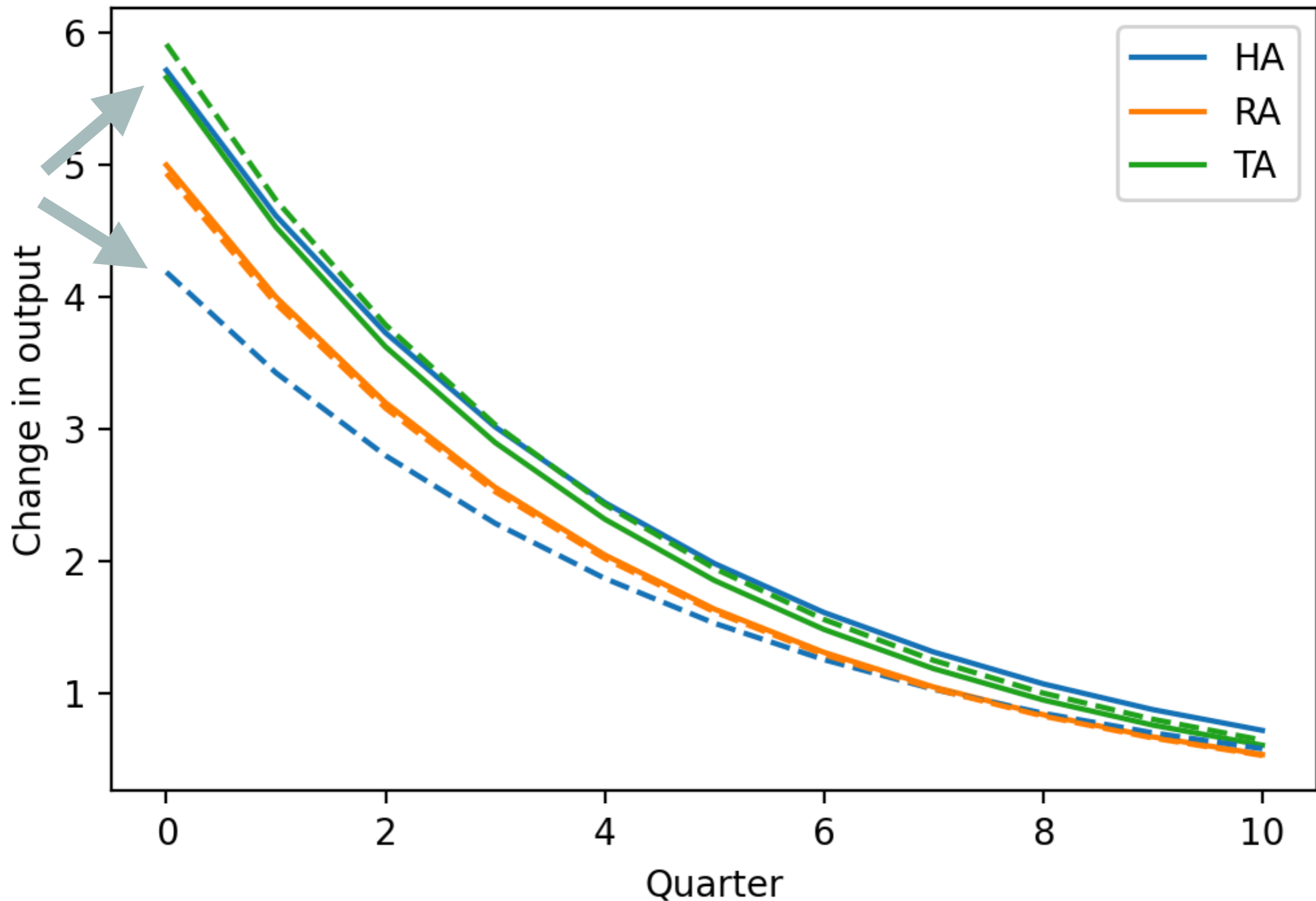
$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & \cdots \\ M_{10} & M_{11} & M_{12} & \cdots \\ M_{20} & M_{21} & M_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \longrightarrow \mathbf{M} = \begin{pmatrix} M_{00} & 0 & 0 & \cdots \\ M_{10} & M_{00} & 0 & \cdots \\ M_{20} & M_{10} & M_{00} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

Redo our analysis with this alternative, myopic \mathbf{M} matrix where households respond to income and taxes only after the fact.

RESULTS: INCOME MYOPIA ONLY MATTERS MUCH FOR HA MODEL!

Change in output, regular (solid) vs. myopic (dashed)



A BIGGER ROLE FOR DEVIATIONS FROM RATIONAL EXPECTATIONS

- In representative-agent models, deviating from rational expectations affects monetary transmission in predictable way
 - if people don't know a cut in real interest rates is coming, they won't react to it!
 - but how much should we rely on people not knowing what interest rates are?
- With large indirect effects, now new mechanisms:
 - knowledge of changes in output (or taxes) makes a big difference even if everyone agrees on the path of rates!
 - opens up a lot! (Farhi and Werning 2019 perhaps first)