Additional questions for 411-3 midterm

Here are some additional review questions for the 411-3 midterm, for which I'm also posting solutions. When combined with last year's midterm and the problem sets, this might be more than you will have time to cover; either these questions or last year's midterm would be a decent guide to this year's midterm.

Problems

Problems 1–3 here are likely more straightforward, and 4–7 are more involved.

1. What is the Gini coefficient of a uniform distribution $[\underline{x}, \overline{x}]$? What about a Gini coefficient of a Pareto distribution with shape parameter α and minimum value $\underline{x}$? How do these depend on parameters?

2. Suppose that the mean of a Pareto distribution is M times the minimum. What is the shape parameter α in terms of M? What if the median is M times the minimum?

3. Suppose that income above \$1 million is given by a Pareto distribution with shape parameter α, and lawmakers impose an additional 1% "millionaire tax" on all income above \$1 million. Among all people who have income of at least \$1 million, what fraction of total income is subject to this tax?[1]

4. Suppose that we are in continuous time, and that the flow rate at which people are born is ϕ times the population, while everyone has a constant flow probability η of dying. Everyone is born with \$1 of wealth and consumes at a rate c out of their wealth, which earns a constant real return of r. Under what conditions does we get a Pareto distribution for wealth, and what is the shape parameter as a function of the parameters above? What is the effect of changes in these parameters on aggregate wealth per capita and on the shape parameter?

5. Consider a hybrid Bewley-Aiyagari model, where the standard incomplete markets model on the household side is embedded in an economy where assets consist of both capital and government bonds. Suppose that the production function is Cobb-Douglas in capital and labor and that the government targets some ratio of bonds-to-output $B/Y \equiv b$. Write equations characterizing steady-state asset market clearing, and derive the first-order effect on real interest rates and the capital-output ratio of an increase in b.

6. Suppose that the $\mathbf{M}$ matrix mapping aggregate after-tax income to aggegate consumption is given by

$$M_{ts} = \begin{cases} 0.5 & t = s, s+1 \\ 0 & t \neq s, s+1 \end{cases}$$

   i.e. that if income increases by dY_s at date s, then consumption increases by $0.5 \cdot dY_s$ at dates s and $s+1$, and is unchanged elsewhere. If income has iid shocks with standard deviation σ, what is the autocorrelation of consumption?

7. How do government spending and taxes impact output in the two-agent New Keynesian model if it is modified so that increased taxes at the margin are only assessed on the "savers", not the hand-to-mouth agents? Is the "balanced budget multiplier" on spending paid for by current taxes still one? Any comments on multipliers more generally?

[1] It's not 100%, since this tax is only assessed on income in a \$1 million+ tax bracket, excluding the first \$1 million of income earned by each income millionaire.