# Investment Hangover and the Great Recession

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*We present a model of investment hangover motivated by the Great Recession. Overbuilding of durable capital such as housing requires a reallocation of productive resources to other sectors, which is facilitated by a reduction in the interest rate. When monetary policy is constrained, overbuilding induces a demand-driven recession with limited reallocation and low output. Investment in other capital initially declines due to low demand, but it later booms and induces an asymmetric recovery in which the overbuilt sector is left behind. Welfare can be improved by ex-post policies that stimulate investment (including in overbuilt capital), and ex-ante policies that restrict investment.*

After 2008, the US economy went through the worst macroeconomic slump since the Great Depression. Real GDP per capita declined from more than $49,000 in 2007 (in 2009 dollars) to less than $47,000 in 2009, and surpassed its pre-recession level only in 2013. The civilian employment ratio, which stood at about 63 percent in January 2008, fell below 59 percent by the end of 2009, and remained below 59.5 percent by the end of 2015.

Recent macroeconomic research emphasizes the boom-bust cycle in house prices—and its effects on financial institutions, firms, and households—as the main culprit for these developments. The collapse of home prices arguably affected the economy through at least two principal channels. First, financial institutions that suffered losses related to the housing market cut back their lending to firms and households (Brunnermeier (2009), Ivashina and Scharfstein (2010)). Second, homeowners who had borrowed against rising collateral values during the boom faced tighter borrowing constraints and had to reduce their outstanding leverage (Eggertsson and Krugman (2012), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017)). Both channels reduced aggregate demand, plunging the economy into a Keynesian recession. The recession was exacerbated by the zero lower bound on the nominal interest rate, also known as the liquidity trap, which restricted the ability of monetary policy to counter demand shocks (Hall (2011), Christiano, Eichen-
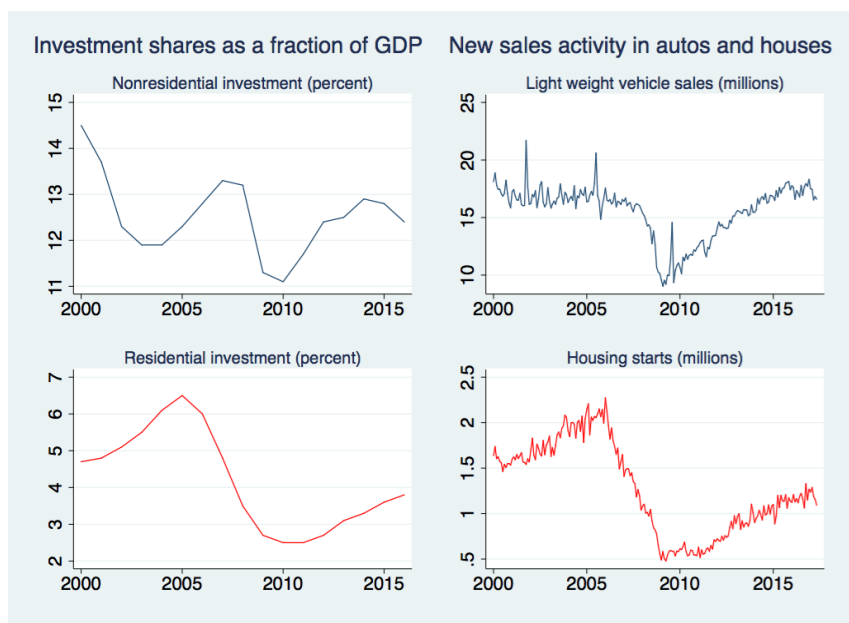
FIGURE 1. THE LEFT PANELS PLOT THE TWO COMPONENTS OF INVESTMENT IN THE US AS A SHARE OF GDP. THE RIGHT PANELS PLOT NEW SALES OF AUTOS AND LIGHT TRUCKS (TOP) AND HOUSING STARTS (BOTTOM). SOURCE: U.S. BUREAU OF ECONOMIC ANALYSIS AND U.S. BUREAU OF THE CENSUS, RETRIEVED FROM ST. LOUIS FED.

baum, Trabandt (2015)).

A growing body of evidence shows that these views are at least partially correct: the financial and the household crises both appear to have played a part in the Great Recession.[1] But these views also face a challenge in explaining the nature of the recovery after the Great Recession. The recovery has been quite asymmetric across components of aggregate private spending. As the right panel of Figure 1 illustrates, nonresidential investment—measured as a fraction of output—almost reached its pre-recession level by 2015, while residential investment remained depressed. One explanation for this pattern is that households were unable to buy homes due to ongoing deleveraging. But the right panel of Figure 1 casts doubt on this explanation: sales of durables not directly related to housing such as cars—which should also be affected by household deleveraging—rebounded strongly while sales of new homes lagged behind. Another potential explanation is that US residential investment generally lags behind in recoveries. This explanation is also incorrect: Leamer (2007) analyzes the post-war recessions in the US and shows that residential investment typically recovers before nonresidential investment and other consumer durables.

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[1] Several recent papers, such as Campello, Graham, and Harvey (2010) and Chodorow-Reich (2014), provide evidence that financial crisis affected firms' investment before 2010. Mian, Rao, Sufi (2013) and Mian and Sufi (2014, 2015) provide evidence that household deleveraging reduced household consumption and employment between 2007 and 2009.
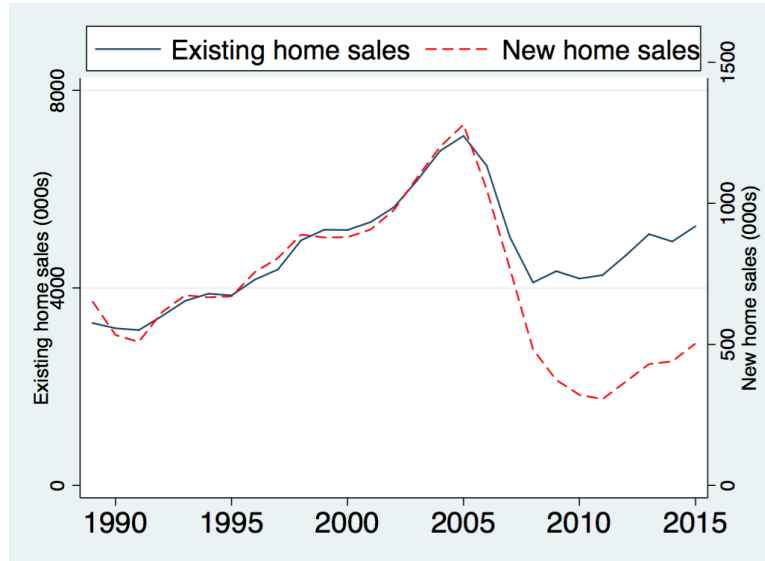
FIGURE 2. THE DASHED CURVE PLOTS THE NEW ONE FAMILY HOMES SOLD IN THE U.S. (SEASONALLY ADJUSTED ANNUAL RATE). SOURCE: U.S. BUREAU OF THE CENSUS, RETRIEVED FROM ST. LOUIS FED. THE SOLID CURVE PLOTS THE EXISTING HOME SALES IN THE U.S. (SEASONALLY ADJUSTED ANNUAL RATE). SOURCE: COPYRIGHT ©2016 "EXISTING HOME SALES PROGRAM." NATIONAL ASSOCIATION OF REALTORS®. ALL RIGHTS RESERVED. REPRINTED WITH PERMISSION.

In this paper, we supplement these two accounts of the Great Recession with a third channel, which we refer to as the *investment hangover*. Our key hypothesis is that there was also an *investment boom* in the housing market in addition to the price boom, which led to an overbuilding of housing capital by 2005. This hypothesis is supported by economic theory as well as empirical evidence. Standard investment theories (e.g., the Q theory) suggest that an asset price boom driven by optimism about asset valuations should also be associated with an investment boom. Moreover, once the valuations are revised downwards, past investment would appear excessive in retrospect—which is what we refer to as overbuilding. Consistent with theory, Figure 1 illustrates a sharp increase in residential investment and housing starts before 2005. Since housing capital is highly durable, it was arguably overbuilt by the Great Recession (see Section IV.B for further empirical support for this hypothesis).

Motivated by our hypothesis, we develop a stylized macroeconomic model to analyze how the economy behaves after overbuilding a durable type of capital—such as housing, structures, or infrastructure (e.g., roads or railroads). Our model's first prediction is that investment in overbuilt capital declines. Intuitively, an excess of initial stock substitutes for new investment. Figure 2 provides evidence from the Great Recession consistent with this prediction. The sales of newly constructed homes, which have historically changed in proportion to the sales of existing homes, fell disproportionately starting around 2005.

Of course, the challenge for a macroeconomic model is to understand why not only housing, but other investment as well as consumption also fell in the Great Recession. In principle, the flow of resources out of housing investment would reduce interest rates, and lead to a boom in other investment and consumption. In our view, monetary policy plays a central role in this aggregate reallocation mechanism. If inflation cannot increase in the short run—an assumption that we maintain—then the real interest rate can fall and counter the demand shock only if monetary policy lowers the nominal interest rate. In practice, many constraints on monetary policy might prevent this from happening. In the aftermath of the Great Recession, monetary policy in developed economies was constrained by the zero lower bound on the nominal interest rate. In economies with fixed exchange rates (e.g., the eurozone), or exogenously determined money supply (e.g., the gold standard), monetary policy is often constrained even if the nominal interest rate is above zero. We show how these types of constraints undermine the aggregate reallocation mechanism. If the initial overbuilding is sufficiently large, then the interest rate does not decline sufficiently and overbuilding induces a demand-driven recession.

Our model shows how a slowdown in the overbuilt sector naturally spills over to other sectors. The recession reduces the return to other types of capital—such as equipment and machines—which were not necessarily overbuilt but are used in the production of overbuilt capital. Thus, other types of investment can also decline, in line with the acceleration principle of investment (Samuelson (1939)), despite the low cost of capital implied by the low interest rate. As the economy decumulates the overbuilt capital, other investment recovers in anticipation of a recovery in output. Through the lens of our model, then, the recession can be roughly divided into two phases. In the first phase, all types of investment decline, generating a severe and widespread slump. In the second phase, investment in overbuilt capital remains low but other investment increases, generating a partial recovery. In the context of the Great Recession, this implies that housing investment is left behind in the recovery, as in Figure 1.

We attempt a quantitative evaluation of the model during the Great Recession. To begin, we provide an accounting exercise that decomposes the decline in output into various components of demand, including residential investment, non-residential investment and consumption. We then calibrate the model, and find that using a housing starts shock from Haughwout et al. (2013) and fairly standard parameter estimates, our model can match the path of housing prices and investment, but does not generate an overall recession. We then augment the model with two other plausible shocks—one to the risk premium and one to household leverage—and show that a calibrated version of the augmented model generates an overall recession. Moreover, the calibrated model matches the demand-side accounting results for the Great Recession reasonably well. The overbuilding shock generates a sizeable cumulative decline in output, in addition to explaining the asymmetric recovery of housing and nonhousing investment.

We investigate the implications of our analysis for policies directed towards regulating investment. A naive intuition would suggest that the planner should not stimulate investment in overbuilt capital, such as housing during the Great Recession, since the problems originate in this sector. We find that this intuition is incorrect: if the reces-

sion is sufficiently severe, then the planner optimally stimulates investment and slows down the decumulation of overbuilt capital. This result is driven by two factors. First, the planner recognizes that raising investment in a demand-driven recession stimulates aggregate demand and employment. In view of these *aggregate demand externalities*, the planner perceives a lower cost of building than does the private sector. The lower cost, by itself, is not a sufficient reason for intervention—the planner also considers the benefits. The second factor is that the return from investment in overbuilt capital need not be low—especially for long-lived capital such as housing or infrastructure. New investment will generate low flow utility in the short run but will be useful in the future. Stimulating investment in overbuilt capital is beneficial because it helps to economize on future investment.

We find that, before the economy enters the liquidation episode, it is optimal for the planner to reduce the accumulation of capital, so as to stimulate investment and aggregate demand during the recession. This result is also driven by aggregate demand externalities, and it holds as long as the agents in our model assign a positive probability to the recession (that is, the planner does not need to fully anticipate the recession to intervene). Our model suggests that the intervention is more desirable for investment in more durable types of capital, because durability is the link by which past investment affects future economic activity. Taken together, our welfare analysis supports policies that intertemporally substitute investment towards periods that feature deficient demand, especially for long-lived capital.

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section I describes the equilibrium in the baseline model in which the economy decumulates the overbuilt capital in a single period. Section II characterizes this equilibrium, and presents our main result that excessive overbuilding induces a recession. Section III considers a variant of the baseline model (with adjustment costs) in which the decumulation is spread over multiple periods, so as to investigate the behavior of investment within the recession. Section IV presents a quantitative evaluation of our model. Section IV.A performs a demand-side accounting of the output decline in the Great Recession, and Section IV.B analyzes the extent to which a calibrated version of our model can replicate the accounting results. Section V analyzes the ex-post and ex-ante policy implications of our analysis using variants of the baseline model. Section VI concludes. The online appendices A, B, and C contain respectively the details of our calibration exercise, extensions of the baseline model, and the omitted proofs.

## A.   *Related literature*

Our paper makes contributions to several strands of the literature. First, we identify the ex-ante overbuilding of housing as an important source of deficient aggregate demand during the Great Recession. A large literature emphasizes other types of demand shocks such as those driven by financial frictions or household deleveraging.[2] Other papers em-

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[2] See also Gertler and Karadi (2011), Midrigan and Philippon (2016), Jermann and Quadrini (2012), He and Krishnamurthy (2014) for quantitative dynamic macroeconomic models that emphasize either banks', firms', or households'

phasize long-run factors that might have lowered demand more persistently (Summers (2013), Eggertsson and Mehrotra (2017), Caballero and Farhi (2017)). Our paper complements this literature and provides an explanation for why residential investment has lagged behind in the recovery.

Another strand of the literature investigates the role of housing during the Great Recession, but often focuses on channels other than overbuilding. Many papers, e.g., Iacoviello and Pavan (2013), emphasize the collateral channel by which house price shocks tighten household borrowing constraints. Boldrin et al. (2016) also emphasize overbuilding, but they do not analyze the resulting deficient demand problem. Instead, they focus on the supply-side input-output linkages by which the slowdown in construction spills over to other sectors.[3]

Second, and more broadly, we illustrate how overbuilding durable capital can trigger a recession. As DeLong (1990) discusses, Hayekian (or liquidationist) views along these lines were quite popular before and during the Great Depression, but were relegated to the sidelines with the Keynesian revolution in macroeconomics. Our paper illustrates how Hayekian and Keynesian mechanisms can come together to generate a recession. The Hayekian mechanism finds another modern formulation in the recent literature on news-driven business cycles. A strand of this literature argues that positive news about future productivity can generate investment booms, occasionally followed by liquidations if the news is not realized (see Beaudry and Portier (2014) for a review). This literature typically generates business cycles without nominal rigidities (see, for instance, Beaudry and Portier (2004), Jaimovich and Rebelo (2009)), whereas we emphasize nominal rigidities and a demand side channel.

In recent and complementary work, Beaudry, Galizia, Portier (BGP, 2017) also investigate whether overbuilding can induce a recession driven by deficient demand. In BGP, aggregate demand affects employment due to a matching friction in the labor market, whereas we obtain demand effects through nominal rigidities. In addition, BGP show how overbuilding increases the (uninsurable) unemployment risk, which exacerbates the recession due to households' precautionary savings motive. We describe how overbuilding exacerbates the recession due to the endogenous response of other types of investment.[4]

Third, our analysis illustrates how a constrained monetary policy, e.g., due to the liquidity trap or exogenous money supply, restricts the efficient reallocation of resources

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financial frictions during the Great Recession.

   [3]A large literature develops quantitative business cycle models with housing, but without focusing on the Great Recession or overbuilding, e.g., Greenwood and Hercowitz (1991), Gervais (2002), Iacoviello (2005), Campbell and Hercowitz (2005), Davis and Heathcote (2005), Fisher (2007), Piazzesi, Schneider, and Tuzel (2007), Kiyotaki, Michaelides, and Nikolov (2011).

   [4]The literature on lumpy investment has also considered the possibility of a hangover (or conversely, pent-up demand), driven by past aggregate shocks that can shift the latent distribution of firms' investment imbalances (see Caballero, Engel, Haltiwanger (1995)). Thomas (2002) argued that the lumpiness, and the associated latent investment distribution, does not affect aggregate investment much once the cost of capital is endogenized. House (2014) clarified that this result is driven by the feature of standard neoclassical models—with or without lumpy investment—that the timing of investment is highly elastic with respect to the changes in cost of capital. However, most empirical evidence suggests that investment timing is not so elastic, especially over short and medium horizons (see Caballero (1999)). As House (2014) also notes, "the key property of the model which generates the irrelevance results—the infinite elasticity of investment demand—is a feature of the models and may not be a feature of reality."

between sectors. A large literature investigates the role of reallocation shocks compared to aggregate activity shocks in generating unemployment (see, for instance, Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Davis and Haltiwanger (1990)). Our paper shows that the constrained monetary policy blurs the line between reallocation and aggregate activity shocks. In our setting, reallocation away from investment in overbuilt capital triggers a Keynesian recession. Moreover, other types of investment also decline earlier in the recession, generating sectoral comovement that resembles an aggregate activity shock. Caballero and Hammour (1996) describe a supply-side channel by which reallocation is restricted because the expanding sectors are constrained due to a hold-up problem.

Fourth, we obtain several positive and normative results for investment when the economy experiences a demand-driven recession. These results apply regardless of whether the episode is driven by overbuilding or some other (temporary) demand shock.[5] On the positive side, we show that investment can decline earlier in the recession, even if the real interest rate remains low and there are no financial frictions, because low aggregate demand lowers the return to investment. This mechanism is reminiscent of the acceleration principle of investment that was emphasized in an older literature (see Clark (1917) or Samuelson (1939)).[6] On the normative side, we show that the private investment decisions during or before the recession are typically inefficient, and characterize the constrained optimal interventions. These results complement a recent literature that analyzes the inefficiencies driven by aggregate demand externalities. Korinek and Simsek (2016) and Farhi and Werning (2016) focus on ex-ante financial market allocations, such as leverage and insurance, whereas we establish inefficiencies associated with physical investment.[7]

## I. Baseline model

The economy is set in infinite discrete time $t \in \{0, 1, ...\}$ with a single consumption good, and three factors of production: a special type of durable capital, $h_t$, other capital, $k_t$, and labor, $l_t$. Our prime example for the special capital is housing, and thus we refer to it also as housing capital. Other examples are structures or infrastructure. For brevity, we also refer to nonhousing capital as "capital." Each unit of housing capital produces one unit of housing services. Capital and labor are combined to produce the consumption good according to a neoclassical technology that we describe below.

Absent shocks, the economy converges to a level of housing capital denoted by $h^*$, which we refer to as the target level (see Eq. (3) below). We analyze situations in

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[5]A growing theoretical literature investigates various aspects of the liquidity trap, but often abstracts away from investment for simplicity (see, for instance, Krugman (1998), Eggertsson and Woodford (2003), Auerbach and Obstfeld (2005), Adam and Billi (2006), Jeanne and Svensson (2007), Werning (2012)).

[6]The mechanism is also present in many other New Keynesian models with capital and constrained monetary policy, but it is not always emphasized. Schmitt-Grohe and Uribe (2017) also show that the liquidity trap can generate an investment slump driven by low return.

[7]A separate literature emphasizes the inefficiencies in physical investment driven by pecuniary externalities (see, for instance, Shleifer and Vishny (1988), Lorenzoni (2008), Stein (2012), Hart and Zingales (2015), He and Kondor (2016), Davila and Korinek (2017)).

which the economy starts with an initial housing capital that exceeds the target, $h_0 > h^*$, which we refer to as *overbuilding*. We interpret the assumption, $h_0 > h^*$, as capturing an unmodeled overbuilding episode that took place before the start of our model. In particular, suppose the (expected) housing demand increased in the recent past relative to its historical level. The economy has built housing capital to accommodate this high level of demand, captured by $h_0$. At date 0, the economy receives news that the high demand conditions are not sustainable. The stock of housing capital needs to adjust to its historical average, captured by $h^*$. Section V introduces an ex-ante period and formalizes this interpretation. We would like to understand how the economy reacts to an excess stock of housing capital.

In our baseline model, we assume that one unit of the consumption good can be converted into one unit of housing or nonhousing capital, or vice versa, without adjustment costs. Thus, the two types of capital evolve according to,

$$
(1) \qquad h_{t+1} = h_t \left( 1 - \delta^h \right) + i_t^h \text{ and } k_{t+1} = k_t \left( 1 - \delta^k \right) + i_t^k.
$$

Here, $i_t^h$ (resp. $i_t^k$) denote housing (resp. nonhousing) investment, and $\delta^h$ (resp. $\delta^k$) denotes the depreciation rate for housing (resp. nonhousing) capital. These assumptions also imply that the relative price of housing capital is constant and normalized to one. Hence, overbuilding in this setting will reduce housing investment without changing house prices. In Sections III and IV, we introduce adjustment costs to housing capital, in which case overbuilding also reduces the price of housing capital.

HOUSEHOLDS. — The economy features a representative household whose problem can be written as,

$$
(2) \qquad \max_{\{l_t, \hat{c}_t, a_{t+1}, i_t^h\}_t} \sum_{t=0}^{\infty} \beta^t \left( u \left( \hat{c}_t - v \left( l_t \right) \right) + u^h \mathbf{1} \left[ h_t \geq h^* \right] \right),
$$
$$
\text{s.t.} \quad \hat{c}_t + a_{t+1} + i_t^h = w_t l_t + \Pi_t + a_t \left( 1 + r_t \right)
$$
$$
h_{t+1} = h_t \left( 1 - \delta^h \right) + i_t^h
$$

The household earns wage income per unit of labor ($w_t$), and receives profits from firms that will be described below ($\Pi_t$). She chooses labor ($l_t$), consumption ($\hat{c}_t$), financial asset holdings ($a_{t+1}$) and housing investment to maximize discounted utility. The functions in her per-period utility, $u \left( \cdot \right), v \left( \cdot \right)$, satisfy the standard regularity conditions. She also receives utility from housing services as captured by the separable term, $u^h \mathbf{1} \left[ h_t \geq h^* \right]$. This is equal to $u^h$ if $h_t \geq h^*$ and zero otherwise, where we take $u^h$ to be a large constant.

Our specification of household preferences in (2) relies on two simplifying assumptions. Households receive a large utility from investing up to $h^*$ but zero marginal utility from additional units. This implies that when $u^h$ is sufficiently large (and the interest rate

is not too negative, $r_{t+1} > -\delta^h$) the household's optimal housing investment is

$$(3) \qquad h_{t+1} = h^*, \text{ which implies } i_t^h = h^* - h_t\left(1 - \delta^h\right).$$

Intuitively, households invest or disinvest so as to reach the target level of housing capital in a single period.[8] In particular, starting with some $h_0 > h^*$, the economy decumulates the excess residential capital in one period. Hence, a period in this setting should be thought of as long as necessary (several years) to adjust the housing capital to its steady-state level. In our extensions with adjustment costs (Sections III and IV), decumulation is spread over multiple periods.

The second simplification in household preferences is the functional form $u\left(\hat{c}_t - v\left(l_t\right)\right)$, which implies that the household's labor supply decision does not depend on its consumption (see Greenwood, Hercowitz and Huffman (GHH, 1988)). Specifically, the optimal labor solves the static problem,

$$(4) \qquad e_t = \max_{l_t} w_t l_t - v\left(l_t\right).$$

Here, $e_t$ denotes households' labor income net of the disutility of labor. We also define $c_t = \hat{c}_t - v\left(l_t\right)$ as *net consumption*. In terms of the net variables, the household solves a standard consumption and savings problem,

$$(5) \qquad \max_{\{c_t, a_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \text{ s.t. } c_t + a_{t+1} + i_t^h = e_t + \Pi_t + a_t\left(1 + r_t\right).$$

The optimal household behavior is summarized by Eq. (3) and problems (4) and (5).

REMARK 1 (Role of the Simplifying Preference Assumptions): GHH preferences provide tractability. In Section V, we show that our main results continue to obtain in a version of the model with separable preferences at date 0, $u\left(c_0\right) - v_0\left(l_0\right)$. The kinked demand for housing services plays a more important role. First, this assumption considerably simplifies the housing investment part of our model, as illustrated by Eq. (3), and allows us to focus on the effect of overbuilding on the remaining equilibrium allocations. Second, the assumption also implies that housing investment in the short run does not react to the changes in the interest rate. With more elastic housing demand, housing investment would qualitatively follow a similar dynamic path as in our model (described in Section II), but its initial decline would be dampened as monetary policy responds by lowering the interest rate. These additional effects are not a major concern for our analysis, because we focus on scenarios in which monetary policy is constrained.

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[8]Housing capital not only provides housing services but it also represents an investment technology. Hence, Eq. (3) also requires the gross interest rate, $1 + r_{t+1}$, to be greater than the gross return (on unutilized houses), $1 - \delta^h$. This is ensured by the required condition, $r_{t+1} > -\delta^h$, which will be the case in equilibrium.

INVESTMENT FIRMS, PRODUCTION FIRMS, AND THE CONSTRAINED INTEREST RATE. — The capital stock of the economy is managed by a competitive investment sector. This sector invests up to the point at which the cost of capital is equated to the net return to physical capital,

$$(6) \qquad\qquad r_{t+1} = R_{t+1} - \delta^k.$$

Here, $R_{t+1}$ denotes the rental rate. The cost of capital is the same as the safe interest rate since there is no uncertainty and no risk premium. In the calibration exercise in Section IV.B, we introduce a wedge between the cost of capital and the safe interest rate, which can be thought of as capturing a risk premium in reduced form. The capital market clearing condition is given by $a_t = k_t$.

Our key ingredient is that monetary policy is constrained so that the nominal interest rate does not react to demand shocks as much as in a real business cycle model. In practice, there are several reasons why the monetary authority might be unable or unwilling to lower the interest rate sufficiently to counter demand shocks. In our baseline analysis, we consider the zero lower bound constraint on the nominal interest rate, $r^n_{t+1} \geq 0$, which appeared to be relevant during the Great Recession (see Online Appendix B.2 for an alternative setting with exogenous money supply). The zero lower bound constraint emerges because cash in circulation yields zero interest in addition to providing households with transaction services.[9] If the nominal interest rate fell below zero, then individuals would switch to hoarding cash instead of holding financial assets. Therefore, monetary policy cannot lower the nominal interest rate (much) below zero. The situation in which the nominal interest rate is at its lower bound is known as the liquidity trap.

Constraints on the nominal interest rate might not affect the real allocations by themselves. However, we also assume that nominal prices are sticky so that a constraint on the nominal rate translates into a constraint on the real rate. For analytical tractability, we assume prices are completely sticky (see Remark 2 below for an interpretation and a discussion of how the results would generalize). This ensures that the real interest rate is also bounded from below,

$$(7) \qquad\qquad r^n_{t+1} = r_{t+1} \geq 0 \text{ for each } t.$$

As we will show, the lower bound on the interest rate plays a central role by creating an upper bound on investment as well as consumption.

We formally introduce nominal price rigidities with the standard New Keynesian model. Specifically, there are also two types of production firms. A competitive final good sector uses intermediate varieties $v \in [0, 1]$ to produce the final output according to the Dixit-Stiglitz technology, $\hat{y}_t = \left( \int_0^1 \hat{y}_t (v)^{\frac{\varepsilon-1}{\varepsilon}} dv \right)^{\varepsilon/(\varepsilon-1)}$ where $\varepsilon > 1$. In turn, a unit mass of monopolistic firms labeled by $v \in [0, 1]$ each produces the variety accord-

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[9]To simplify the notation and the exposition, however, we do not explicitly model money or its transaction services in the main text. Online Appendix B.2 analyzes a version of the model with these features.

ing to, $\hat{y}_t(v) = F(k_t(v), l_t(v))$, where $F(\cdot)$ is a neoclassical production function with standard regularity conditions.

Each monopolist has a preset and constant nominal price, $P_t(v) = P$ for each $v$. This assumption implies that the aggregate price level is also constant, thereby formalizing the bound in (7).[10] The assumption also implies that monopolists are symmetric: they face the same real price (equal to one) and they choose the same level of inputs and outputs subject to an aggregate demand constraint. In particular, the representative monopolist's problem can be written as:

$$(8) \qquad \Pi_t = \max_{k_t, l_t} F(k_t, l_t) - w_t l_t - R_t k_t \text{ s.t. } F(k_t, l_t) \le \hat{y}_t.$$

REMARK 2 (Role of the Price Stickiness Assumption): The extreme price stickiness captures in reduced form a situation in which inflation is sticky in the *upward* direction during the decumulation episode. In practice, this type of stickiness could be driven by nominal rigidities at the micro level (in the goods market or the labor market), or by constraints on monetary policy against creating inflation. It is also worth emphasizing that making the prices more flexible at the micro level does not necessarily circumvent the bound in (7). In fact, if monetary policy cannot credibly commit to creating inflation (e.g., due to an inflation targeting policy regime), then limited price flexibility exacerbates the bound in (7). This is because the negative output gap during the zero lower bound episode exerts a downward pressure on inflation. As the inflation (and expected inflation) falls, the real interest rate increases and the demand shortage becomes more severe (see Remarks 1-3 in Korinek and Simsek (2016) for further discussion).

EFFICIENT BENCHMARK AND MONETARY POLICY. — In the equilibria we analyze, the monopolists who solve problem (8) will find it optimal to meet all of their demand. Thus, the output satisfies $\hat{y}_t = F(k_t, l_t)$. With GHH preferences, we also find it convenient to work with *the net output*, that is, output net of labor costs, $y_t = \hat{y}_t - v(l_t)$. The net output is determined by the net aggregate demand, $y_t = c_t + i_t^k + i_t^h$, which in turn is determined by monetary policy as well as other factors.

To describe monetary policy, we first define the efficient benchmark at some date $t$ as the continuation allocation that maximizes households' welfare subject to the feasibility constraints (and given the state variables, $k_t, h_t \ge h^*$). Online Appendix C.1 shows that the efficient benchmark solves a standard neoclassical planning problem. With GHH preferences, the efficient employment maximizes the net output in every period. We define

$$(9) \qquad L(k_t) \in \arg\max_{\tilde{l}} F(k_t, \tilde{l}) - v(\tilde{l}) \text{ and } S(k_t) = F(k_t, L(k_t)) - v(L(k_t))$$

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[10]Alternatively, we could also assume that firms set their prices at every period mechanically according to a predetermined inflation target, that is, $P_t(v) = P(1 + \pi)^t$ for some $\pi \ge 0$. This formulation yields a very similar bound as in (7) and results in the same economic trade-offs. We normalize the inflation target to zero so as to economize on notation.

as respectively the efficient (or the supply-determined) level of the labor supply and net output.

Since the price level is fixed, we assume that monetary policy focuses on stabilizing employment and output (analogous to a Taylor rule). In our setting, this corresponds to replicating the (statically) efficient allocations in (9) subject to the constraint in (7).[11] Specifically, monetary policy follows the interest rate rule,

$$(10) \qquad r_{t+1}^n = r_{t+1} = \max\left(0, r_{t+1}^*\right) \text{ for each } t \geq 0.$$

Here, $r_{t+1}^*$ is recursively defined as the natural interest rate that obtains when the employment and the net output at date $t$ are given by (9) and monetary policy follows the rule in (10) at all future dates. This policy is constrained efficient in our environment as long as monetary policy cannot use forward guidance: that is, it cannot commit to setting future interest rates beyond the current period.

DEFINITION 1: *The        equilibrium       is       a       path       of       allocations,* $\left\{h_t, k_t, l_t, \hat{c}_t, c_t, i_t^h, i_t^k, \hat{y}_t, y_t\right\}_t$*, and real prices and profits,* $\{w_t, R_t, r_{t+1}, \Pi_t\}_t$*, such that the households and firms choose allocations optimally as described above, the interest rate is set according to* (10)*, and all markets clear.*

## II.  Investment overhang

We next characterize the equilibrium and establish our main result that excessive overbuilding triggers a demand-driven recession. We start by establishing the properties of equilibrium within a period.

LEMMA 1:  *(i) If $r_{t+1} > 0$, then $y_t = S\left(k_t\right)$, $l_t = L\left(k_t\right)$, and $R_t = S'\left(k_t\right)$.*

*(ii) If $r_{t+1} = 0$, then the net output is below the efficient level, $y_t \leq S\left(k_t\right)$, and is determined by net aggregate demand, $y_t = c_t + i_t^k + i_t^h$. The labor supply is below its efficient level, $l_t \leq L\left(k_t\right)$, and is determined as the unique solution to,*

$$(11) \qquad y_t = F\left(k_t, l_t\right) - v\left(l_t\right) \text{ over the range } l_t \in \left[0, L\left(k_t\right)\right].$$

*The rental rate of capital is given by $R_t = R\left(k_t, l_t\right) \equiv \frac{v'(l_t)}{F_l(k_t, l_t)} F_k\left(k_t, l_t\right)$, where the function $R\left(k_t, l_t\right)$ is strictly decreasing in $k_t$ and strictly increasing in $l_t$ over the range $l_t \in \left[0, L\left(k_t\right)\right]$ with $R\left(k_t, L\left(k_t\right)\right) = S'\left(k_t\right)$.*

Part (i) describes the case in which the interest rate is positive and monetary policy replicates the efficient outcomes. Part (ii) describes the case in which monetary policy is constrained by the zero lower bound. In this case, the economy experiences a recession with low net output and employment.

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[11]In our setting, the equilibrium without nominal rigidities would also feature monopoly distortions, which should ideally be corrected by targeted policies such as monopoly subsidies. To simplify the notation, we ignore this distinction and assume the monetary policy attempts to correct for monopoly distortions as well as the distortions due to nominal rigidities.

For either case, the result also characterizes the rental rate of capital, which determines the return to investment. To understand these results, consider monopolists' factor demands, captured by the optimality conditions for problem (8),

$$(12) \qquad (1 - \lambda_t) \, F_k \, (k_t, l_t) = R_t \text{ and } (1 - \lambda_t) \, F_l \, (k_t, l_t) = w_t.$$

Here, $\lambda_t \geq 0$ denotes the Lagrange multiplier on the demand constraint in (8). Combining the second equation with the household optimality condition implies that the Lagrange multiplier is equal to the labor wedge,

$$(13) \qquad \lambda_t = \tau_t \equiv 1 - v' \, (l_t) \, / F_l \, (k_t, l_t) \, .$$

If the interest rate is positive, then employment is at its efficient level and the labor wedge is zero. In this case, Eq. (12) implies that capital earns its marginal contribution to supply. If instead the interest rate is zero, then employment is below its efficient level and the labor wedge is positive. In this case, capital earns a lower return characterized by combining Eqs. (12) and (13). Intuitively, low aggregate demand also lowers the return to capital, which will play a central role in our analysis of the investment response in Section III.

Lemma 1 implies further that the capital stock is bounded from above,

$$(14) \qquad k_{t+1} \leq \bar{k} \text{ for each } t, \text{ where } S' \, (\bar{k}) - \delta^k = 0.$$

Here, the upper bound $\bar{k}$ is the level of capital that delivers a net return of zero absent a demand shortage. Investing beyond this level would never be profitable given the lower bound to the cost of capital implied by (7), as well as the possibility of a demand shortage. Intuitively, only so many projects can be undertaken without violating the lower bound on the interest rate.

DYNAMIC EQUILIBRIUM WITH INVESTMENT OVERHANG. — We next characterize the dynamic equilibrium under the assumption that the economy starts with too much housing capital,

$$(15) \qquad h_0 = (1 + b_0) \, h^*, \text{ where } b_0 > 0.$$

Here, $b_0$ parameterizes the degree of past overbuilding. Eq. (3) then implies,

$$(16) \qquad i_0^h = h^* - \left(1 - \delta^h\right) h_0 = \left(\delta^h - b_0 \left(1 - \delta^h\right)\right) h^*.$$

Note that housing investment at date 0 is below the level required to maintain the target, $i_0^h < \delta^h h^*$. Thus, overbuilding represents a negative shock to housing investment. The equilibrium depends on how the remaining components of aggregate demand, $i_0^k$ and $c_0$, respond to this shock.

To characterize this response, we solve the equilibrium backwards. Suppose the economy reaches date 1 with $h_1 = h^*$ and some capital level $k_1 \leq \bar{k}$. Since the housing capital has already adjusted, the zero lower bound does not bind in the continuation equilibrium, that is, $r_{t+1} > 0$ for each $t \geq 1$. Consequently, monetary policy replicates the efficient benchmark starting date 1 given $h_1 = \bar{h}$ and $k_1 \leq \bar{k}$. Online Appendix C.1 shows that the solution converges to a steady-state $(k^*, l^*, y^*, c^*)$, characterized by

(17) $$S'(k^*) - \delta^k = 1/\beta - 1, \text{ and}$$
$$l^* = L(k^*), y^* = S(k^*), c^* = S(k^*) - \delta^k k^* - \delta^h h^*.$$

The initial consumption is given by $c_1 = C(k_1)$, for an increasing function $C(\cdot)$.

Next consider the equilibrium at date 0. The key observation is that aggregate demand is bounded from above due to the constraint on the interest rate. We have already seen in Eq. (14) that capital is bounded from above, which implies that nonhousing investment is bounded,
$$i_1^k \leq \bar{k} - (1 - \delta^k) k_0.$$

Likewise, consumption is bounded by the Euler equation at the zero interest rate,

(18) $$c_0 \leq \bar{c}_0, \text{ where } u'(\bar{c}_0) = \beta u'(C(\bar{k})).$$

Combining the bounds in (14) and (18) with the demand shock in (16), the aggregate demand (and output) at date 0 is also bounded,

(19) $$y_0 \leq \bar{y}_0 \equiv \bar{k} - (1 - \delta^k) k_0 + \bar{c}_0 + (\delta^h - b_0(1 - \delta^h)) h^*.$$

The equilibrium depends on the comparison between the maximum demand and the efficient level, i.e., whether $\bar{y}_0 < S(k_0)$. This in turn depends on whether the amount of overbuilding $b_0$ exceeds a threshold level,

(20) $$\bar{b}_0 \equiv \frac{\bar{k} - (1 - \delta^k) k_0 + \bar{c}_0 + \delta^h h^* - S(k_0)}{(1 - \delta^h) h^*}.$$

PROPOSITION 1 (Overbuilding and the Demand-driven Recession):   *Suppose $b_0 > 0$.*
*(i) If $b_0 \leq \bar{b}_0$, then, the date 0 equilibrium features,*

$$r_1 \geq 0, y_0 = S(k_0) \text{ and } l_0 = L(k_0).$$

*(ii) If $b_0 > \bar{b}_0$, then, the date 0 equilibrium features a demand-driven recession with,*

$$r_1 = 0, k_1 = \bar{k}, y_0 = \bar{y}_0 < S(k_0) \text{ and } l_0 < L(k_0).$$

*Moreover, the net output $y_0$ and the labor supply $l_0$, as well as the actual output, $\hat{y}_0 =*
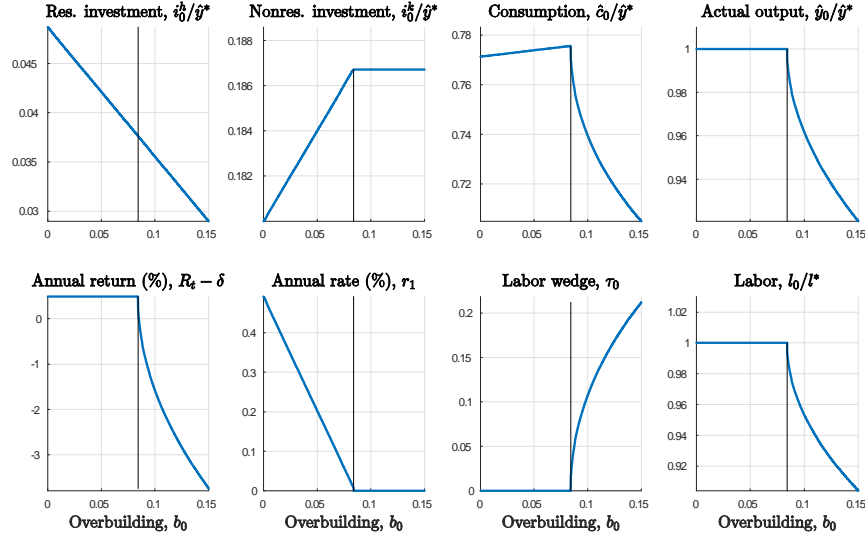
FIGURE 3. DATE 0 EQUILIBRIUM VARIABLES AS A FUNCTION OF THE INITIAL OVERBUILDING $b_0$ (MEASURED AS A FRACTION OF THE STEADY-STATE RESIDENTIAL CAPITAL STOCK, $h^*$).

$y_0 + v(l_0)$, *are decreasing in* $b_0$.

*In either case, the continuation allocations starting at date* 1 *feature positive interest rates and solve a neoclassical planning problem. The economy converges to a steady state* $(k^*, l^*, y^*, c^*)$ *given by* (17).

Part (i) describes the equilibrium for the case in which the initial overbuilding is not too large. In this case, the economy does not experience a demand-driven recession. Low investment in housing capital is offset by a reduction in the interest rate and an increase in nonhousing investment and consumption, leaving the output and employment determined by productivity. The left part of the panels in Figure 3 (the range corresponding to $b_0 \leq \overline{b}_0$) illustrate this outcome for a particular parameterization (and starting with $k_0 = k^*$).

Part (ii), our main result, characterizes the case in which the initial overbuilding is sufficiently large. In this case, the reduction in aggregate demand due to low housing investment is large enough to plunge the economy into a demand-driven recession. The lower bound on the interest rate prevents the nonhousing investment and consumption from expanding sufficiently to pick up the slack, which leads to low output and employment. Figure 3 illustrates this result. Greater overbuilding triggers a deeper recession.

COMPARATIVE STATICS OF THE RECESSION. — When is a given amount of overbuilding, $b_0$, more likely to trigger a demand-driven recession? As illustrated by Eq. (20), factors that reduce aggregate demand at date 0, such as a higher discount factor $\beta$ (that lowers

$\bar{c}_0$), increase the incidence of the demand-driven recession in our setting. More generally, frictions that reduce aggregate demand during the decumulation phase, such as household deleveraging or financial crisis, are complementary to our mechanism. Intuitively, this is because the zero lower bound represents a nonlinear constraint on monetary policy. A demand shock that lowers the interest rate leaves less slack for monetary policy, increasing the potency of other demand shocks such as overbuilding.

Eq. (20) illustrates further that overbuilding of the two types of capital is complementary in terms of triggering a demand-driven recession: greater initial level of nonhousing capital stock $k_0$ increases the incidence of a demand-driven recession. A high level of $k_0$ affects the equilibrium at date 0 through two channels. First, it reduces nonhousing investment at date 0 and lowers aggregate demand—similar to a high level of $h_0$. Second, it increases the efficient output, $S(k_0)$, and makes a demand shortage more likely.

A distinguishing feature of housing capital is its high durability relative to many other types of capital. In Online Appendix B.1, we consider a slight variant of the model to investigate whether high durability is conducive to triggering a demand-driven recession. The extension features two types of housing capital, one more durable (i.e., depreciates more slowly) than the other. The analysis reveals that, controlling for the total amount of overbuilding in both types of capital, overbuilding durable capital is more likely to trigger a demand-driven recession. Intuitively, depreciation helps to "erase" the overbuilt capital, reducing the impact of past overbuilding on future aggregate demand. Since durable capital depreciates more slowly, once overbuilt it tends to stay around for longer and reduce aggregate demand by a larger amount. This suggests that overbuilding is a greater concern when it hits durable capital such as housing, structures, or infrastructure, as opposed to less durable capital such as equipment or machinery.[12]

OTHER CONSTRAINTS ON MONETARY POLICY. — For concreteness, we focus on the zero lower bound constraint on monetary policy. While the ZLB is relevant for the Great Recession, monetary policy can be constrained for many other reasons. In economies with fixed exchange rates (such as the eurozone), monetary policy is often constrained because it is linked with the policies of other countries. In economies under the gold or silver standard, which have been historically common, monetary policy is constrained since the money supply is linked to the quantity of precious metals. As should be clear from our analysis, our main result (as well as our results in subsequent sections) continues to apply when monetary policy is constrained for reasons other than the zero lower bound.

Online Appendix B.2 illustrates this point by deriving a version of Proposition 1 in an environment in which the money supply follows an exogenous path. In this setting, the interest rate is determined by the money supply and household liquidity preferences. Since these forces are largely exogenous, the interest rate does not decline sufficiently to meet the decline in aggregate demand. Consequently, excessive overbuilding triggers

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[12]A previous literature has empirically investigated whether the overbuilding of information technology (IT) equipment during the boom years of late 1990s and 2000 might have contributed to the 2001 recession in the US (see Desai and Goolsbee (2004) and the references therein).
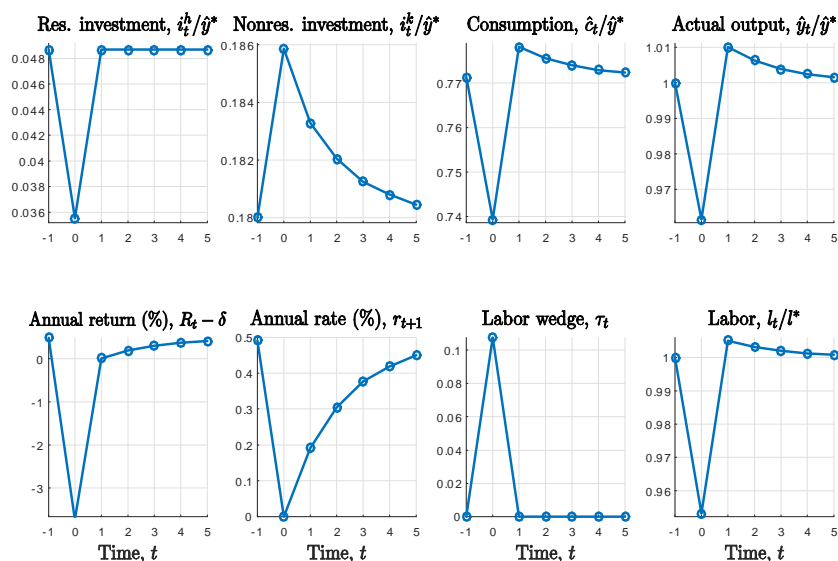
FIGURE 4. THE EVOLUTION OF EQUILIBRIUM VARIABLES OVER TIME, STARTING WITH $b_0 > \bar{b}_0$.

a demand-driven recession, as in our baseline analysis. In fact, the recession is often more severe because, while the interest rate during the recession declines to zero in the baseline model, it can remain above zero when the money supply is exogenous. Our investment hangover mechanism can thus shed light on some historical business cycles other than the Great Recession.[13]

DYNAMICS OF THE SLUMP. — So far, we have been agnostic about the equilibrium before the start of our model (date −1). Our results apply generally as long as the ex-ante equilibrium leads to $h_0$ that sufficiently exceeds $h^*$. To describe how the macroeconomic

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[13]A prime example is the American business cycle of 1879-1885, which is often linked with an investment cycle in railroads—a highly durable type of capital that constituted a sizeable fraction of aggregate investment at the time. Fels (1952) provides a narrative of the episode and describes many of the effects predicted by our model, including the accelerator effects on other capital as well as an asymmetric recovery in which the overbuilt railroad sector is left behind (see Section III). He writes:

> Construction of railroads was the principal factor in the upswing. The number of railroad miles built rose spectacularly from 2665 in 1878 to 11569 in 1882...The downswing gathered momentum slowly in 1883. The decline in railroad construction not only eliminated the jobs of many workers directly employed in railroad building but also spread depression to other industries. As one would expect from the theory of the acceleration principle, the iron and steel industry was particularly affected... Nevertheless, contraction gradually came to an end and gave way to weak revival in the course of 1885... But railroad-building by itself, which was to experience a great boom in 1886 and especially 1887, was at its lowest ebb at the time of the cyclical upturn. Thus, the upturn occurred not because of the behavior of railroad construction but in spite of it.

aggregates change over time, we now suppose that the economy is initially at a steady state in which household preferences are associated with a higher target level of housing, $h^{*,old} = h_0$. The equilibrium allocations at date $-1$ are determined according to this steady state. At the beginning of date 0, there is an unexpected and permanent shock to household preferences that lowers the target level of housing to $h^* < h_0 = h^{*,old}$. The equilibrium allocations from date 0 onwards are then characterized by Proposition 1.

Figure 4 plots the resulting dynamic equilibrium. Housing investment, employment, and output decline during the slump, but nonhousing investment increases. The recession is confined to the overbuilt sector. This is a counterfactual prediction that we revisit in the next section. Note that the aftermath of the slump (date 1 onwards) features a neoclassical adjustment to the steady state.

REMARK 3 (Consumption Response and the Multiplier): The effect of overbuilding on consumption during the slump is nuanced. While it always increases net consumption, $c_t$, in view of the Euler equation (18), it might actually reduce actual consumption, $\hat{c}_t = c_t + v(l_t)$, in view of the decline in employment, $l_t$ (see Figure 4). The latter channel is driven by the GHH utility function, $u(\hat{c} - v(l))$, which generates a Keynesian multiplier effect even though we work with a representative household. We view this feature of the model as capturing in reduced form the multiplier effect that would emerge in more realistic variants of our model with heterogenous households and borrowing constraints.[14] In Online Appendix A, we follow Auclert and Rognlie (2017) and show that the first-order approximation to the multiplier in a period is given by the inverse of the labor wedge in that period, $1/\tau_t$ (cf. Eq. (13)). Hence, the multiplier is especially powerful in the neighborhood of the supply-determined region that features a zero labor wedge. In our calibration exercise in Section IV.B, we introduce appropriately calibrated labor taxes to ensure that the multiplier is smaller and in line with the empirical estimates from the recent macroeconomic research.

## III. Investment response and asymmetric recovery

In our model so far, overbuilding (that is large enough to trigger a recession) reduces housing investment, employment, and output. However, as Figure 4 illustrates, it increases nonresidential investment, which is not consistent with what happens in major recessions. This feature of the model is driven by the stylized features of our baseline analysis. In this section, we develop a version of the model with housing adjustment costs, which can naturally generate a recession in which investment in both types of capital decline.

The analysis is motivated by Figure 4, which shows that overbuilding reduces the net return to nonhousing capital during the recession (see also Lemma 1). One could expect

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[14]In our NBER working paper, we extend the model by introducing additional households that have high marginal propensities to consume (MPC) out of income. We show that, if there are sufficiently many high-MPC households, then overbuilding generates a decline in net consumption during the recession (as well as actual consumption). Intuitively, the low output in the recession lowers all households' incomes, which in turn reduces aggregate consumption due to the high-MPC households.

these low returns to translate into low nonresidential investment. This does not happen in the baseline model, however, because the current capital, $k_0$, is already predetermined and the next period's capital, $k_1$, is associated with full recovery. Introducing housing adjustment costs enables us to spread the recession over multiple periods, and thus to investigate the investment response during the recession.

We now assume investing $i_t^h$ reduces the household utility by $\frac{1}{2}\psi\left(i_t^h - i^{h,*}\right)^2$. Thus, the adjustment costs take a quadratic form centered around the steady-state level of investment, $i^{h,*} = \delta^h h^*$. The parameter $\psi$ captures the level of the adjustment costs. The household then solves the following analog of problem (2),[15]

$$(21) \quad \max_{\{l_t, \hat{c}_t, a_{t+1}, i_t^h\}_t} \quad \sum_{t=0}^{\infty} \beta^t \left( u\left(\hat{c}_t - v\left(l_t\right)\right) + u^h \mathbf{1}\left[h_t \geq h^*\right] - \frac{1}{2}\psi\left(i_t^h - i^{h,*}\right)^2 \right),$$

$$\text{s.t.} \qquad \hat{c}_t + a_{t+1} + i_t^h = w_t l_t + \Pi_t + a_t\left(1 + r_t\right),$$

$$h_{t+1} = h_t\left(1 - \delta^h\right) + i_t^h.$$

To analyze the equilibrium, let $Q_t$ denote the Lagrange multiplier on the last equation (that describes the evolution of housing capital) divided by the Lagrange multiplier on the budget constraint. Note that $Q_t$ measures the shadow price of housing capital at the end of period $t$ (in terms of the consumption good). Taking the first order conditions for $i_t^h$ and $\hat{c}_t$, we obtain a relatively standard q-theory equation,

$$(22) \qquad i_t^h - i^{h,*} = \frac{u'\left(c_t\right)}{\psi}\left(Q_t - 1\right), \text{ where } i_t^h = h_{t+1} - h_t\left(1 - \delta^h\right).$$

The first order condition for $h_{t+1}$, together with the Euler equation, implies the asset pricing equation, $Q_t = \delta_{t+1} + \frac{1-\delta^h}{1+r_{t+1}}Q_{t+1}$, where $\delta_{t+1}$ denotes a subgradient of the kinked function, $u^h \mathbf{1}\left[h_t \geq h^*\right]$. Under our assumption that $u^h$ is sufficiently large, this can be rewritten as a complementary slackness condition,

$$(23) \qquad Q_t \geq \frac{1-\delta^h}{1+r_{t+1}}Q_{t+1}, h_{t+1} \geq h^* \text{ and one of the inequalities hold as equality.}$$

When $h_{t+1} > h^*$, the marginal utility from housing capital is zero, and thus the price is equal to the value of the part of housing capital that will remain nondepreciated in the next period. The remaining equilibrium conditions are the same as in the previous

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[15]The results are generally similar when we specify adjustment costs in terms of final goods instead of utility. We choose the latter formulation so that the adjustment costs do not directly affect aggregate demand, which facilitates the calibration exercise in the next section.
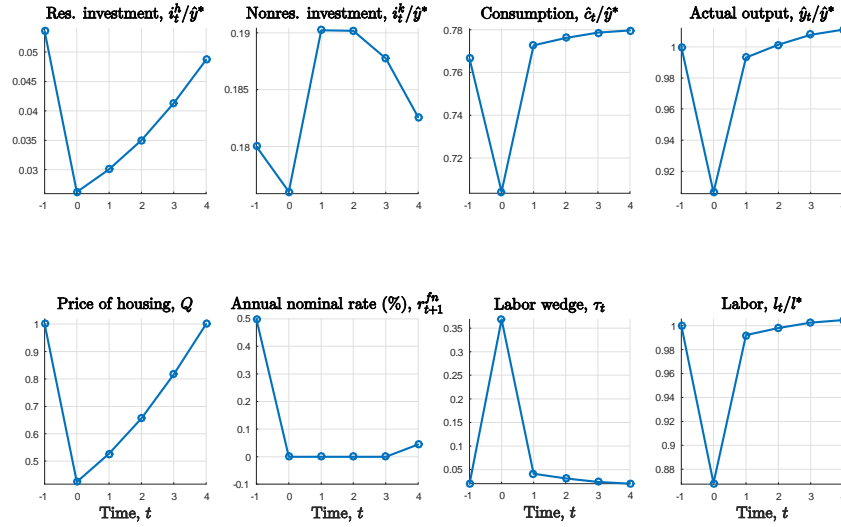
section, and can be written as,

(24)
$$u'\left(c_{t}\right) = \beta \left(1 + r_{t+1}\right) u'\left(c_{t+1}\right),$$
$$R\left(k_{t+1}, l_{t+1}\right) - \delta^{k} = r_{t+1},$$
$$y_{t} = F\left(k_{t}, l_{t}\right) - \upsilon\left(l_{t}\right) = c_{t} + k_{t+1} - \left(1 - \delta^{k}\right) k_{t} + i_{t}^{h},$$
$$y_{t} \le S\left(k_{t}\right), r_{t+1} \ge 0 \text{ and one of the inequalities hold as equality.}$$

where $S\left(k\right)$ is defined by Eq. (9) and $R\left(k, l\right) = \frac{\upsilon'(l)}{F_{l}(k,l)} F_{k}\left(k, l\right)$ (see Lemma 1). Here, the first equation is the Euler equation, the second equation is the optimality condition for investment firms, and the third equation is the aggregate resource constraint. The last equation captures the monetary policy rule in (10), which we have rewritten in complementary slackness form.

The equilibrium corresponds to the path that solves Eqs. $(22 - 24)$ for each $t$. While all variables are jointly determined, it is useful to think of Eqs. (22) and (23) as determining the optimal path of housing prices and investment, $\left\{Q_{t}, i_{t}^{h}\right\}_{t=0}^{\infty}$, that brings the economy from $h_{0}$ to $h^{*}$. In view of adjustment costs, residential investment can (and typically does) remain below its steady-state level for multiple periods. The equations in (24) then determine the periods over which the economy is in a liquidity trap (if any), as well as the equilibrium allocations during and after the liquidity trap.

Figure 5 illustrates the dynamic equilibrium for a particular parameterization. As before, date $-1$ corresponds to the initial steady state in which the target level of housing

capital is $h^{*,old} = h_0$. A sufficiently large amount of overbuilding generates a liquidity trap throughout the decumulation phase. The difference with the previous section is that the decumulation is now spread over multiple periods. In this example, the decumulation is completed in four periods, that is, $h_t > h^*$ for $t \in \{0, 1, 2, 3\}$ and $h_4 = h^*$.[16] More strikingly, nonresidential investment follows a nonmonotonic path during the recession: it declines in the second period but increases afterwards.

To understand this nonmonotonic investment response, first note that when $r_{t+1} = 0$ the investment optimality condition in (24) can be rewritten as,

$$R(k_{t+1}, l_{t+1}) - \delta^k = 0.$$

Recall that the return function $R(\cdot)$ is decreasing in the capital stock $k_{t+1}$ and increasing in employment $l_{t+1}$. Note also that, in the liquidity trap, employment is determined by aggregate demand and output: formally, $l_{t+1}$ is the solution to $y_{t+1} = F(k_{t+1}, l_{t+1}) - \upsilon(l_{t+1})$. The optimality condition then implies that, when the aggregate output at date $t + 1$ is expected to be greater, firms invest more at date $t$, and obtain a greater capital stock at date $t + 1$. Our model thus implies a version of the acceleration principle of investment, which posits that investment is proportional to expected changes in output.[17]

Next recall that overbuilding triggers a demand-driven recession and reduces output. When the reduction in output is sufficiently large, investment in the earlier phase of the recession declines due to (our version of) the acceleration principle. Intuitively, the recession lowers the return to capital, as depicted in Figure 3, and firms optimally respond to this low return to capital. In later periods, aggregate demand and output gradually increase in anticipation of the eventual recovery. As this happens, the low cost of capital becomes the dominant factor for nonhousing investment. Consequently, the economy starts reaccumulating capital, and in fact exits the slump with the maximum level of capital $\bar{k}$ as in the previous section.

It follows that the recession in our model can be divided into two phases. In the first phase, captured by period 0 in the model, both types of investment fall. This induces a particularly severe recession with low output and employment. In the second phase, housing investment remains low whereas nonhousing investment recovers. The investment response also raises aggregate demand. The second phase of the recession in our model represents a partial and asymmetric recovery in which the housing sector is left behind, similar to the aftermath of the Great Recession (see Figure 1).

## IV. A quantitative evaluation

We next turn to a quantitative evaluation of the investment overhang mechanism in the context of the Great Recession. We first provide an accounting exercise that decomposes

[16]A period in this version of the model should be thought of as lasting shorter than in the baseline model. In fact, we calibrate the example so that the outcomes over dates $t \in \{0, 1, 2, 3\}$ in Figure 5 can be directly compared to date $t = 0$ in Figure 4.

[17]See Eckaus (1953) for a review of the early literature on the acceleration principle, and our NBER working paper version for a discussion of how our mechanism differs from this literature.

the decline in output into various components of demand. We find that residential invest-ment accounts for a sizeable fraction of the output shortfall during the Great Recession, and that its effects are more persistent than those of nonresidential investment and con-sumption. We then ask whether a calibrated version of our model can reproduce these effects when we feed it with direct measures of housing overbuilding (obtained from ex-ternal sources). As we show, the overbuilding shock helps to explain the variables related to the housing market. It is not sufficiently strong to trigger the recession by itself, but it also helps to explain output and employment when combined with two other shocks that arguably lowered demand in recent years.

### A.    Accounting for the decline in output during the Great Recession

We first construct a slightly modified measure of output that is consistent with our model. Specifically, we exclude from GDP net exports as well as government consump-tion and investment—as these variables do not have counterparts in our model. We also exclude housing services (i.e., rents), which our model accounts for as part of household utility as opposed to output. Hence, our measure of output is given by $y_t = c_t + i_t + i_t^h$, where $c_t$ denotes private consumption (excluding housing services), $i_t$ denotes private nonresidential investment, and $i_t^h$ denotes private residential investment (excluding com-missions). The top left panel in Figure 6 illustrates the evolution of $y_t$ (in real prices) together with the actual GDP.

We evaluate the changes in output and its components relative to a trend level, denoted by $y_t^*$, that can be thought of as approximating the level of output that would obtain absent demand shocks. The typical approach is to apply a smoothing (and detrending) procedure to historical output series. We believe this approach is not appropriate for the Great Recession since the postwar data does not have any other episodes in which the interest rate hit the zero lower bound. Guided by theory, we instead construct $y_t^*$ by interpolating the actual output between 2007 and 2016. Our approach implicitly assumes the economy featured zero output gaps in 2007 and 2016, and negative output gaps in between, since nominal interest rates declined to zero in 2008 and remained at zero until December 2015. While this assumption is strong, it leads to a relatively transparent accounting exercise. The right panel of Figure 6 plots the output relative to trend, $y_t/y_t^*$. The lower left panels of Figure 6 plot each component of output as a fraction of trend output, respectively $i_t^h/y_t^*$, $i_t/y_t^*$, and $c_t/y_t^*$. Note that there is a sharp fall and gradual recovery in output as well as each of its components.

We next decompose the shortfall in output according to the following equation,

$$\Delta \frac{y_t}{y_t^*} = \Delta \frac{i_t^h}{y_t^*} + \Delta \frac{i_t}{y_t^*} + \Delta \frac{c_t}{y_t^*}, \text{ where } \Delta x_t = x_t - x_{2007}.$$

The right panel of Figure 6 visualizes this decomposition. From top to bottom, the dif-ference between curves illustrates respectively the contribution of residential investment, $\Delta \frac{i_t^h}{y_t^*}$, investment, $\Delta \frac{i_t}{y_t^*}$, and consumption, $\Delta \frac{c_t}{y_t^*}$. Residential investment accounts for about 25 percent of the output decline in 2009. Nonresidential investment and consumption ac-
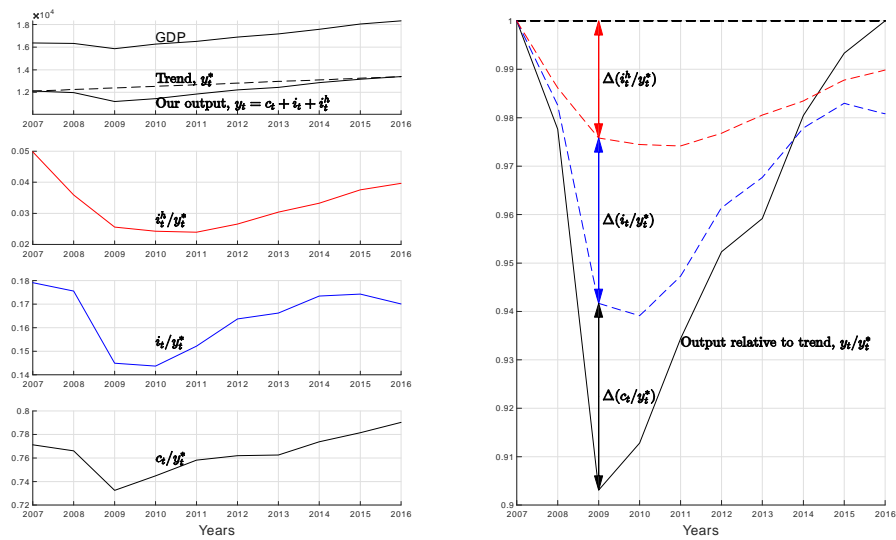
FIGURE 6. THE TOP LEFT PANEL PLOTS OUR CONCEPT OF OUTPUT AS WELL AS ACTUAL GDP (FOR THE US) IN 2009 PRICES. THE BOTTOM LEFT PANELS PLOT THE COMPONENTS OF OUTPUT AS A FRACTION OF TREND OUTPUT. THE RIGHT PANEL SHOWS OUTPUT AS A FRACTION OF TREND OUTPUT, AND DECOMPOSES THE SHORTFALL IN OUTPUT INTO CONTRIBUTIONS COMING FROM ITS COMPONENTS.

count for respectively 35 and 40 percent of the decline. Since residential investment has a delayed recovery, its relative contribution grows over time and exceeds 50 percent by 2012. In contrast, since consumption recovers rapidly, its contribution declines over time and in fact becomes negative by 2014. The relative contribution from nonresidential investment remains roughly stable during the slump.

### B. Calibrated model predictions for the Great Recession

We next calibrate the model and ask to what extent housing overbuilding can explain the accounting results. For a reasonable calibration, we first generalize the baseline model (with housing adjustment costs) in two ways. These generalizations do not strengthen our overbuilding mechanism. Their only role is to ensure that the underlying neoclassical model is roughly in line with certain aspects of the data.

The first generalization is to make a distinction between the nominal risk-free interest rate and the return to capital, which are identical in our baseline model (for simplicity). In the data, the risk-free interest rate is much lower than the return to capital. To accommodate this fact, we introduce a constant risk-premium wedge, $\phi \geq 0$ (later, we will make this time-varying). In particular, the return to capital is now given by $r_{t+1} = r_{t+1}^f + \phi$, where $r_{t+1}^f$ denotes the real risk-free interest rate. We also introduce a constant inflation rate denoted by $\pi \geq 0$ (see Footnote 10 in Section I for a microfoundation), so that $r_{t+1}^f = r_{t+1}^{fn} - \pi$, where $r_{t+1}^{fn}$ denotes the nominal risk-free interest rate. With these assumptions, the return to capital is related to the nominal interest rate according to, $r_{t+1} = r_{t+1}^{fn} + \phi - \pi$. As before, monetary policy controls the nominal interest rate, $r_{t+1}^{fn}$, which in turn affects the required return to capital. The zero lower bound constraint, $r_{t+1}^{fn} \geq 0$, continues to imply a (possibly non-zero) lower bound on the return to capital. More specifically, the constrained monetary policy rule in (10) is now replaced with,

$$(25) \qquad r_{t+1} = \max \left( \phi - \pi, r_{t+1}^* \right) \text{ for each } t \geq 0.$$

Note that the parameters $\phi$ and $\pi$ do not affect the analysis except for changing the threshold return of capital at which the economy enters the liquidity trap region. We calibrate the inflation rate ($\pi$) as 2 percent per year based on the recent inflation data from the US. We also calibrate the risk premium ($\phi$) as 7.5 percent per year so that, when the real risk-free rate is equal to 1 percent (its historical average), the return to capital is equal to 8.5 percent, which is in line with the estimates in Poterba (1998).

The second generalization concerns the Keynesian multiplier in the baseline model. As we describe in Remark 3, the GHH preferences generate a multiplier effect, which is (to a first order approximation) proportional to the inverse of the labor wedge, $1/\tau_t$. This multiplier is particularly strong in the neighborhood of the frictionless neoclassical equilibrium with zero labor wedge. We introduce labor taxes into the model so as to weaken and parameterize this multiplier effect. Specifically, the monopolist's problem

(8) is now replaced with,

$$(26) \qquad \Pi_t = \max_{k_t, l_t} F\left(k_t, l_t\right) - \frac{1}{1 - \tau^l} w_t l_t - R_t k_t \text{ s.t. } F\left(k_t, l_t\right) \leq \hat{y}_t.$$

Here, $w_t$ denotes the after-tax wage as before and $\frac{1}{1-\tau^l} w_t$ denotes the pre-tax wage. The tax revenues are rebated lump sum to the representative household. The analysis of the equilibrium with labor taxes closely parallels the baseline analysis and is completed in Online Appendix A. In the supply determined region, the labor wedge is equal to the tax parameter, that is, $\tau_t = \tau^l$. We calibrate $\tau^l = 2/3$ so that the implied multiplier in the neighborhood of the supply-determined region is approximately $1/\tau^l = 3/2$, which is in line with the empirical estimates from the recent literature (see, for instance, Nakamura and Steinsson (2014)).

BENCHMARK WITH PURE OVERBUILDING SHOCKS. — We next present a benchmark calibration with overbuilding but no other shocks. The economy starts in January 2008. The period length is two years. These choices are guided by the observation that nonresidential investment fell in 2008 and 2009, and started to recover in 2011 (see Figure 6). In our model, nonresidential investment (with sufficiently strong demand shocks) falls in the first period of the recession and recovers in subsequent periods (see Figure 5). Making the period length two years enables us to match the timing of the recovery in the data and in the model without introducing additional ingredients.[18] We describe the parameters that depend on the period length (such as $\beta$, $\delta$, $r_{t+1}^f$) in terms of their yearly values for ease of interpretation. In our numerical calculations, we transform the parameters appropriately so that each period is two years.

We work with constant elasticity functional forms for output and the disutility of labor,

$$F\left(k, l\right) = \left(\alpha k^{(\varepsilon-1)/\varepsilon} + (1-\alpha) l^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \text{ and } \upsilon\left(l\right) = \frac{1}{1+\nu} l^{1+\nu}.$$

We set the elasticity of substitution between capital and labor, $\varepsilon = 0.5$, based on the meta-analysis of Chirinko (2008). We set the Frisch elasticity of the labor supply, $1/\nu = 0.4$, based on the meta-analysis of Reichling and Whalen (2012).

We set $\beta$ so that the steady-state level of return to capital ($r^*$) is equal to 8.5 percent, in line with the estimates in Poterba (1998). With the risk premium ($\phi$) set at 7.5 percent, this also implies that the steady-state real interest rate is equal to 1 percent, in line with its historical average. We then set $\alpha$ and $\delta$ to ensure the ratio of capital to yearly output is 165 percent, and the ratio of nonresidential investment to output is 18 percent, consistent with the NIPA data in 2007 using our concept of output. These choices imply that the output share of capital in the model is around 32 percent.

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[18]We could also match the timing of the recovery by introducing adjustment costs to nonresidential investment and calibrating the adjustment cost parameter appropriately. We decided against this approach to keep the model as parsimoniuous as possible.

| Params | Description | Calibration | Target | Source |
|--------|-------------|-------------|--------|--------|
| $\beta$ | Discount factor | 0.92 | Ave $K$ return $= 0.085$ | Poterba (1998) |
| $\phi$ | Risk premium | 0.075 | Ave ff-pce, 03–07 | |
| $\pi$ | Inflation | 0.02 | Ave pce infl, 03–07 | |
| $\epsilon$ | $K - L$ elasticity | 0.5 | | Chirinko (2008) |
| $\alpha$ | Share parameter | 0.26 | Match $k/y = 1.65$ | NIPA 2007 |
| $\delta$ | Depreciation rate | 0.11 | Match $i/y = 0.18$ | NIPA 2007 |
| $\tau^l$ | Labor tax | 2/3 | Implied multiplier $= 1.5$ | Nakamura and Steinsson (2014) |
| $1/\theta$ | EIS | 0.1 | Ave micro & macro meta | Havranek (2015) |
| $1/\nu$ | Frisch labor supply | 0.4 | Median of meta-analysis | Reichling and Whalen (2012) |
| $\delta^h$ | Res depreciation rate | 0.034 | Match $i^h/y = 0.05$ | NIPA 2007 |
| $h_0$ | 07 housing stock (mil) | $129\kappa$ | | Haughwout et al (2013) |
| $h^*$ | 07 housing need (mil) | $125.6\kappa$ | | Haughwout et al (2013) |
| $\kappa/y_{-1}$ | \$ per mil h, over GDP | 0.011 | Match $h_0/y = 1.48$ | NIPA 2007 |
| $\psi$ | Res adj cost | $150u'(c^*)$ | Decum. in 6 yr | Haughwout et al (2013) |
| $\phi^{new} - \phi$ | Risk premium shock | 0.03 | Takes economy to ZLB | |
| $\{\gamma_t\}$ | Discount rate shock | (series) | $\Delta c$ from $MPC \times$ h shock | Mian, Rao, Sufi (2013) |

TABLE 1—CALIBRATION PARAMETERS

We work with constant elasticity preferences, $u(c) = (c^{1-\theta} - 1)/(1 - \theta)$. We set the elasticity of intertemporal substitution at $1/\theta = 0.1$, based on the average of the macro and the micro estimates obtained in the meta-analysis by Havranek (2015). Our final calibration results in Section IV.B are robust to allowing for higher levels of $1/\theta$.

We calibrate the amount of overbuilding based on the analysis in Haughwout et al. (2013). They estimate that, after adjusting for demographics, the trend rate of growth of the number of households since the mid-1990s should have been around 1.17 million per year. They also estimate the depreciation from the current stock of housing units should have been around 230 thousand units per year. This suggests that the economy needed about 1.4 million new houses per year since mid-1990s. Their Figure 2.8 provides a measure of oversupply by plotting the cumulative housing production since 1995 relative to the predicted trend of 1.4 million per year. This analysis illustrates that the economy produced many more houses relative to what was needed (based on historical patterns), with the difference peaking at around 3.4 million in mid-2007. They further show that this measure of overbuilding is quite consistent with an alternative measure that they obtain from the Census data on housing vacancies (see Online Appendix A for details).

Based on this analysis, we set

$$(27) \qquad\qquad h_0 = h^{*,old} = 129\kappa \text{ and } h^* = 125.6\kappa,$$

where $\kappa$ can be thought of as the average value of a million housing units. We set $\kappa$ to ensure that the ratio of aggregate housing capital to (yearly) output in the old steady-state is 148 percent. This ratio is consistent with NIPA data in 2007, and it implies that the average value of a housing unit in 2007 is around 140 thousand dollars (in 2009 prices). To see the magnitude of the overbuilding shock, note that 3.4 million excess housing
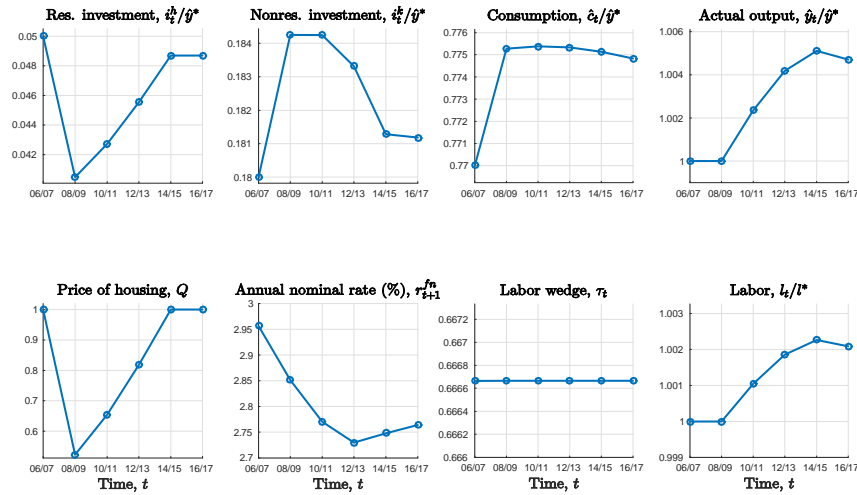
FIGURE 7. BENCHMARK CALIBRATION WITH OVERBUILDING BUT NO OTHER DEMAND SHOCKS.

units corresponds to about 480 billion dollars (in 2009 prices), which amounts to about 4 percent of our measure of output in 2007. Note also that our calibration is rather conservative, since it focuses only on the overbuilding of housing units (excluding, for instance, overbuilding in consumer durables related to housing, as well as overbuilding in the size or the quality of the average home).

We set the housing depreciation rate, $\delta^h$, so that the residential investment to output ratio in the old steady state is equal to 5 percent. We also set the housing adjustment cost parameter, $\psi$, so that decumulation is completed in three periods (or 6 years), that is, $h_t > h^*$ for $t \in \{0, 1, 2\}$ and $h_3 = h^*$. This is consistent with Figure 2.8 in Haughwout et al. (2013), which suggests the overbuilt capital would be worked off in about five years. Table 1 summarizes the parameters used in our analysis.

Figure 7 illustrates the results from our benchmark calibration exercise with pure overbuilding shocks. The residential investment to output ratio falls to 4 percent and gradually reverts to a level that is slightly below 5 percent. This is qualitatively consistent with the behavior of the residential investment series in Figure 6. Quantitatively, residential investment in the data falls by a greater magnitude and does not really recover to 5 percent. We offer two explanations for this discrepancy. First, as we describe above, our calibration is rather conservative and does not consider all potential sources of overbuilding. Second, and more importantly, the collapse of house prices might also have reduced the demand for housing going forward (e.g., due to pessimism about the housing market or credit constraints in the mortgage market). Consistent with this explanation, Haughwout et al. (2013) find that the household formation rate in recent years has been much lower than the rate that they predict based on historical trends and demographics (see

Online Appendix A). Hence, some of the ongoing weakness in the housing market is likely to be driven by unusually low demand. We leave this additional demand shock out of our calibration exercise since it is difficult to quantify.

The model also generates about 50 percent decline in the price of housing capital followed by a recovery. This is qualitatively consistent with the dynamics of house prices in practice, but the price declined less in the data than in the model. This discrepancy obtains in part because we assume the required return on housing capital is the same as for nonhousing capital.[19] In practice, the discount rate for housing capital is likely to be lower, which would make the price of housing depend relatively more on its long-term value and lead to a smaller price decline during the decumulation episode (see Eq. (23)).

Figure 7 also illustrates that, while the overbuilding shock helps to explain the variables related to the housing market, it is not sufficiently large to generate a recession by itself. The interest rate declines by 0.2 percentage points, which is not sufficient to trigger a recession. Note, however, that the decline in the interest rate is quite persistent. This suggests that, if the overbuilding shock is combined with other demand shocks, it could lead to a drag on aggregate output and employment. We verify this in the next subsection.

OVERBUILDING AND RISK PREMIUM SHOCKS. — In our benchmark calibration, the decline in the interest rate triggers an investment boom and mitigates the recession (see Figure 7). In practice, however, the US economy did not experience an investment boom even though interest rates have been unusually low for many years (see Figure 6). To fix this aspect of the model, we introduce a shock to the risk premium parameter, $\phi$. Specifically, we replace the lower bound in (25) with,

$$r_{t+1} = \max\left(\phi_{t+1} - \pi, r^*_{t+1}\right) \text{ for each } t \geq 0,$$

where $\phi_{t+1}$ denotes the risk premium between dates $t$ and $t + 1$. We suppose $\phi_{t+1} = \phi^{new} \geq \phi$ during the periods, $t \in \{0, 1, 2, 3\}$, that correspond to zero interest rates in the data. We calibrate the size of the shock ($\phi^{new} - \phi$) as 3 percent per year so that it puts the economy to the zero lower bound, but it does not reduce output by itself (similar to Midrigan and Philippon (2016)). We also suppose that $\phi_{t+1}$ gradually declines back to $\phi$ over periods $t \geq 4$.[20]

Figure 8 illustrates the results from this exercise. The red (dashed) line illustrates the case with risk premium shocks but without overbuilding. The $\phi$ shock lowers the risk-free rate to zero, but it does not affect the equilibrium allocations by itself. The blue (solid) line illustrates the joint effect of overbuilding and the risk premium shocks. Starting from near-zero interest rates, the overbuilding shock leads to roughly a 2 percentage

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[19]Even after we introduce a risk premium, we continue to assume there is a single rate, $r_t$, that determines the return to housing capital as well as nonhousing capital (see Eq. (25)). A version of the model with multiple rates of return with heterogeneous risk premia would be more realistic but would take us too far from our baseline analysis.

[20]The exact path by which $\phi_{t+1}$ reverts back to its old level affects the path of interest rates but does not matter for the equilibrium allocations. This is because, by period 4, the decumulation is already completed and the outcomes are supply-determined even if $\phi_{t+1} = \phi^{new}$. In this region, the monetary policy increases the nominal interest rate to counter the declines in $\phi_{t+1}$.
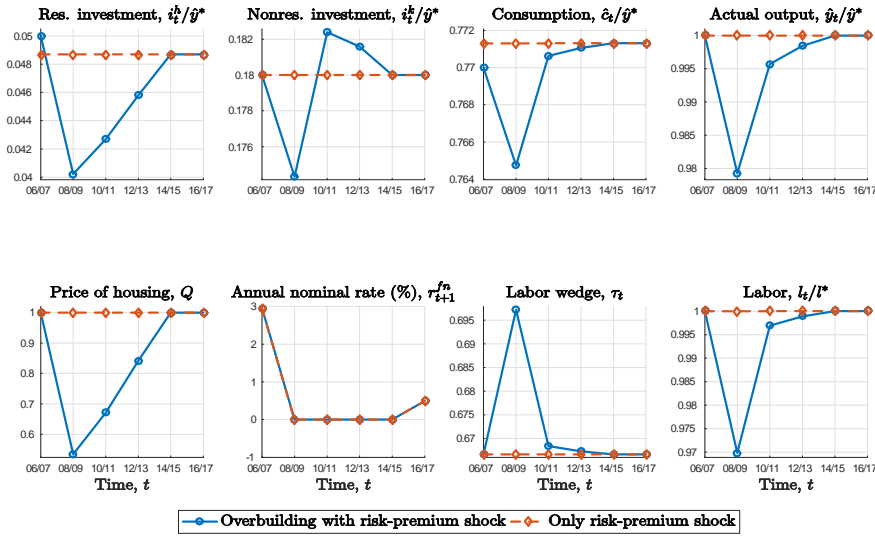
FIGURE 8. CALIBRATION WITH OVERBUILDING AND RISK-PREMIUM SHOCKS.

point reduction in output in the first two years (2008/2009), as well as a cumulative reduction of about 5.3 percentage points of yearly output. To understand where this decline comes from, recall that our overbuilding shock is about 4 percent of yearly output. After accounting for depreciation that naturally erases some overbuilt capital (see Online Appendix B.1), the cumulative shortfall in residential investment (relative to its new steady-state level) is about 3.5 percent of yearly output. With a Keynesian multiplier of 1.5, a back-of-the-envelope calculation would suggest a cumulative decline of 5.2 percentage points, which is close to the decline in our calibration.[21]

OVERBUILDING WITH RISK PREMIUM AND DELEVERAGING SHOCKS. — The risk premium shock ensures that overbuilding affects output and employment. The model also generates a decline in consumption and investment in the first period, but at much smaller levels than those observed in the data (see Figure 6). To fix these aspects, we introduce an additional shock that we interpret as capturing household deleveraging in reduced form. Specifically, we replace the Euler equation with,

$$u'(c_t) = (1 + \gamma_t) \beta (1 + r_{t+1}) u'(c_{t+1}) \text{ for each } t,$$

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[21]This calculation is not exact since there are also the investment accelerator effects that lower investment (and the capital stock) in the initial period and raise it in subsequent periods. The cumulative demand effect of the accelerator is close to zero, and thus, the back-of-the-envelope calculation is roughly accurate.
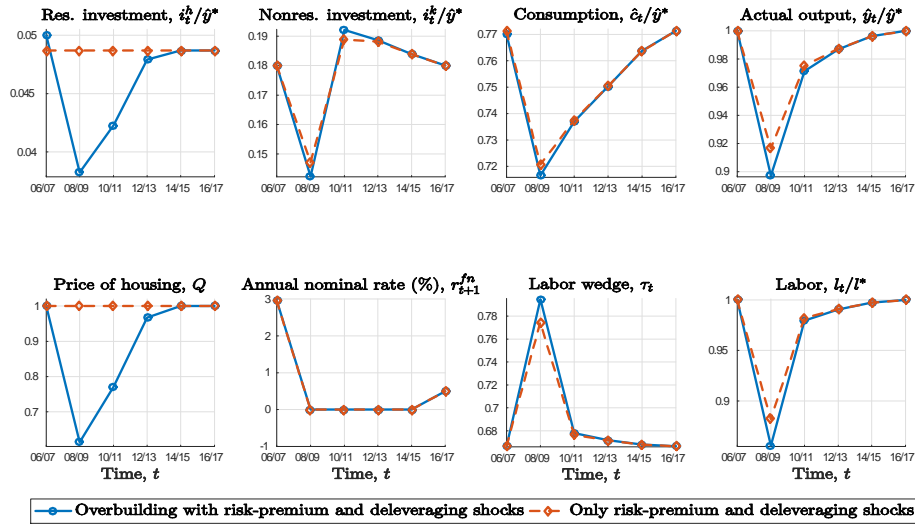
FIGURE 9. FINAL CALIBRATION WITH OVERBUILDING, RISK PREMIUM, AND DELEVERAGING SHOCKS.

for an exogenous sequence $\{\gamma_t\}_{t=0}^{\infty}$. A positive $\gamma_t$ reduces aggregate (net) consumption at time $t$, which can be thought of as capturing household deleveraging. We calibrate $\{\gamma_t\}_{t=0}^{\infty}$ based on the analysis in Mian, Rao, Sufi (2013). Specifically, we combine their estimate for the MPC out of housing net worth with the decline in housing net worth since 2006 to calculate an implied decline in consumption for each year between 2008 and 2015. We then set the sequence $\{\gamma_t\}$ to generate the same consumption declines in periods $t \in \{0, 1, 2, 3\}$ of the model starting with the steady-state consumption in all periods.

Figure 7 plots the results of this exercise. The blue (solid) line is our final calibration that incorporates all three shocks. The red (dashed) line is a comparison benchmark that features the deleveraging and the risk premium shocks without overbuilding. Similar to the previous exercise, the overbuilding shock generates an additional 2 percentage points reduction in output in the first two years (2008/2009) as well as a cumulative reduction of about 4.7 percentage points of yearly output. The three shocks together match the decline in aggregate output as well as its decomposition reasonably well.

Figure 10 illustrates this point by replicating the accounting exercise in Section IV.A in the calibrated model environment. The model-implied decline in investment-output ratio is almost as large as in the data even though we did not introduce a shock (such as the financial crisis) that exclusively lowers investment in the first period. This suggests that the accelerator could be a contributing factor to low investment during the Great Recession. The model-implied decline in consumption is also close to the level observed in the data, which supports the deleveraging mechanism. The contribution of residential investment to the decline in output grows over time in the model as in the data. On
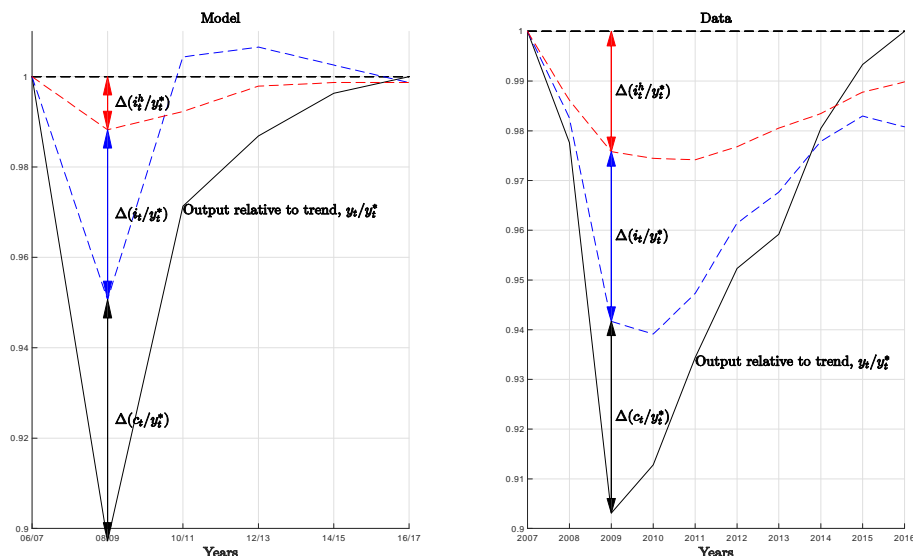
FIGURE 10. THE LEFT PANEL DECOMPOSES THE SHORTFALL IN OUTPUT IN THE CALIBRATED MODEL INTO CON-
TRIBUTIONS COMING FROM ITS COMPONENTS. THE RIGHT PANEL, WHICH IS FROM FIGURE 6, ILLUSTRATES THE
SAME DECOMPOSITION EXERCISE IN THE DATA.

the other hand, the model generates too fast a recovery in nonresidential investment
compared to the data, which also induces too fast a recovery in output. Hence, the model
cannot fully explain the weakness in nonresidential investment in recent years.

We take away four main conclusions from the quantitative evaluation. First, in a
demand-side accounting exercise, residential investment accounts for a relatively size-
able and persistent fraction of the decline in output. Second, the overbuilding shock
(which we measure directly from external data sources) helps to understand the decline
in residential investment, as well as the decline in house prices. Third, when combined
with a risk premium shock that lowers the interest rate to zero, the overbuilding shock
generates a cumulative decline in output roughly equal to 5 percentage points of yearly
output. Fourth, overbuilding, risk premium, and deleveraging shocks can together match
the demand-side accounting results for the earlier phase of the slump reasonably well.

## V.  Policy implications

We next investigate the welfare implications of our analysis. Since our model features
a liquidity trap, several policies that have been discussed in the literature are also rele-
vant here.[22] We skip a detailed analysis of these policies for brevity. Instead, we focus

[22]In particular, welfare can be improved with unconventional monetary policies as in Eggertsson and Woodford (2003),
or unconventional tax policies as in Correia et al. (2013). Once we modify the model appropriately to include government

on constrained policy interventions directed towards controlling investment (housing and nonhousing), which plays the central role in our analysis. We first discuss ex-post policies by which the government can improve welfare once the overbuilding is realized. We then discuss ex-ante policies that the government can implement (prior to date 0) as a precaution.

The policy implications are driven by *aggregate demand externalities*, which are best illustrated by Figure 3 in the baseline setting. In the region $b_0 \geq \bar{b}_0$, increasing the initial stock of housing, $b_0$, does not change the initial net consumption, $c_0$, which is a sufficient statistic for welfare (because it also takes into account labor costs). That is, starting the economy with more housing capital (or conversely, destroying some housing capital) neither raises nor lowers welfare. Intuitively, giving one unit of housing capital to a household raises her welfare (see Eq. (28) below), but it also lowers housing investment. This in turn reduces aggregate demand and employment, and reduces other households' welfare. In the baseline setting, these demand externalities are so strong that they completely undo the direct value of housing capital.

The externalities are very powerful in part because of the multiplier embedded in GHH preferences (see Remark 3). To provide a more transparent cost-benefit analysis for policy interventions, in this section we work with a slight modification of the baseline model in Section I (all of the results also hold in the baseline setting). Suppose at date 0, and only at this date, households' preferences over consumption and labor are given by the separable form, $u(c_0) - v_0(l_0)$, as opposed to the GHH form, $u(\hat{c}_0 - v(l_0))$. With a slight abuse of terminology, we use $c_0$ to denote consumption at date 0 as opposed to net consumption, and $y_0 = F(k_0, l_0)$ to denote output at date 0 as opposed to net output. We also abstract away from adjustment costs so that the competitive equilibrium decumulates the excess capital in a single period. Lemma 2 in Online Appendix B.3 establishes that a sufficiently high level of overbuilding triggers a demand-driven recession also in this setting.

### A. Ex-post policies: Slowing down disinvestment

A natural question in this environment concerns the optimal government policy regarding housing investment. On the one hand, since overbuilding is associated with housing, it might sound intuitive that the planner should not interfere with the decumulation of this type of capital. On the other hand, policies that support the housing market have been widely used during and after the Great Recession (see Berger et al. (2016) for an evaluation of some of these policies). We next formally analyze the desirability of these types of policies.

We start by revisiting the representative household's equilibrium trade-off for housing investment, which provides a useful benchmark for the planner's trade-off. Imagine a household who already invested up to the target level, $h_1 = h^*$, and who is considering investing an additional unit. Online Appendix B.3 defines the value function, $W_0(h_1)$,

spending, welfare can also be improved by increasing government spending during the recession as in Werning (2012) and Christiano, Eichenbaum, and Rebelo (2011).

for this household and shows that,

$$(28) \qquad \frac{d_+ W_0\,(h_1)}{dh_1}\big|_{h_1=h^*} = u'\,(\bar{c}_0)\left(\frac{1-\delta^h}{1+r_1}-1\right) < 0.$$

Here, $\frac{d_+ W_0(h_1)}{dh_1}$ denotes the right derivative, and the inequality follows since $r_1 = 0$. The household assigns a positive value, $1 - \delta^h$, to the excess unit of housing capital: Even though she does not receive any flow utility in the short run, she will benefit from the nondepreciated housing in the future. Nonetheless, she chooses $h_1 = h^*$ in equilibrium because the benefit is lower than the private cost of capital.[23]

Next consider a constrained planner who can fully determine housing investment at date 0, but cannot interfere with the remaining market allocations either at date 0 or in the future. Online Appendix B.3 defines the value function, $W_{0,pl}\,(h_1)$, for this constrained planner and shows that,

$$(29) \qquad \frac{d_+ W_{0,pl}\,(h_1)}{dh_1}\big|_{h_1=h^*} = u'\,(\bar{c}_0)\left(\left(1-\delta^h\right)-(1-\tau_0)+\frac{dc_0}{dh_1}\tau_0\right).$$

Here, $\tau_0 > 0$ is the labor wedge, which captures the severity of the demand shortage (as in the baseline model). Comparing Eqs. (28) and (29) illustrates that the (direct) social benefit of building is the same as the private benefit, $1 - \delta^h > 0$. However, the social cost is lower, $1 - \tau_0 < 1$, which leads to the following result.

PROPOSITION 2 (Slowing Down Disinvestment): *Consider the equilibrium charac- terized in Lemma 2. There exists $\tilde{b}_0$ such that, if $b_0 > \tilde{b}_0$, then the planner chooses a higher level of housing investment than the target level, $h_{1,pl} > h^*$.*

The planner recognizes that housing investment increases aggregate demand and em- ployment. This is socially beneficial, and the benefits are captured by the labor wedge, $\tau_0$, because employment is below its efficient level. Thus, the demand externalities lower the social cost of building. The reduced cost, by itself, does not create sufficient ratio- nale for intervention—the planner also compares the cost with the benefit. Proposition 2 shows that the planner intervenes as long as the initial overbuilding is sufficiently large (since greater overbuilding increases the labor wedge). Eqs. (28) and (29) suggest fur- ther that this is more likely if the overbuilt capital is more durable, so that $1 - \delta^h$ is higher. Intuitively, durable capital—such as housing—has a relatively high value, even if it is overbuilt in the short run, because it helps to economize on future investment.[24]

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[23]The right derivative, $\frac{d_+ W_0(h_1)}{dh_1}$, is strictly less than zero at the optimal housing level, $h_1 = h^*$, since the household preferences in (2) feature a kink at this level.

[24]Eq. (29) illustrates an additional benefit of investing in durable capital, captured by the nonnegative term, $\frac{dc_0}{dh_1}\tau_0$ (see Online Appendix B.3). Intuitively, bringing the nondepreciated part of the capital to date 1 creates a future wealth effect that raises consumption not only at date 1, but also at date 0, which further increases employment. This channel is reminiscent of the forward guidance policies that create a similar wealth effect by committing to low interest rates in the future. In fact, increasing $h_1$ also lowers the future interest rate, $r_2$, in our setting. Note, however, that future output

Our model thus provides a rationale for policies that support housing investment during an investment hangover. In practice, the planner can do this by increasing housing demand, e.g., with mortgage subsidies or modifications, or by increasing housing supply, e.g., with construction subsidies. Both types of policies can internalize the inefficiency in our model. Note, however, that the demand side policies tend to increase house prices, whereas the supply side policies tend to decrease them. The demand side interventions might be more appropriate if one considers additional ingredients, such as financial frictions, that are left out of our analysis.

To isolate the trade-offs, we focused on a planner who can only influence housing investment. In practice, the policymakers can use various other tools to fight a demand-driven slump. Eq. (29) would also apply in variants of the model in which the planner optimally utilizes multiple policies. In those variants, the equation would imply that the planner should stimulate housing investment as long as she cannot substantially mitigate the demand shortage (i.e., lower the labor wedge, $\tau_0$) by using only the other feasible policies. This prediction is arguably applicable to various developed economies in recent years, e.g., the US and Europe, that have featured zero nominal interest rates with low employment and output despite utilizing various stimulus policies.

## B.    Ex-ante policies: Restricting investment

We next analyze whether the planner can improve welfare via ex-ante interventions. To this end, consider the baseline model with an ex-ante period, date $-1$. Suppose also that the economy can be in one of two states at date 0, denoted by $s \in \{H, L\}$. State $L$ is a low-demand state in which the target level of housing capital is $h^*$ as before (and the planner has no tools for ex-post intervention). State $H$ is a high-demand state in which the utility function in (2) is modified so that the target level of housing capital is $\left(1 + \lambda^H\right) h^*$ for some $\lambda^H > 0$. Let $\pi^H \in (0, 1)$ denote the ex-ante probability of the high-demand state at date 0. The economy starts with $h_{-1} = \left(1 + \lambda^H\right) h^*$ and $k_{-1} = k^*$.

The model can be thought of as capturing a situation in which housing demand has recently increased relative to its historical level, and the economy has already adjusted to this new level. However, there is a possibility that the current state is not sustainable and the housing demand will revert back to its historical level. We also envision that $\pi^H$ is large, so that the representative household believes the high-demand state is likely to persist, but also that $\pi^H < 1$ so that there is room for precautionary policies.

We first characterize the choice of $h_0$ and $k_0$ in the competitive equilibrium, which we then compare with the constrained efficient allocations. The preferences in (2) imply that the opportunity cost of consuming housing services below target is very large. Consequently, households invest in housing capital according to their demand in state $H$, that is, $h_0 = \left(1 + \lambda^H\right) h^*$. The degree of overbuilding in state $L$ is now endogenized, $b_0 = \lambda^H$.[25] Nonhousing investment, $k_0$, is in turn determined by a standard optimality

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remains efficient in our model, $y_1 = S\left(k_1\right)$, whereas it exceeds the efficient level in environments with forward guidance (Werning, 2012).

[25]The feature that overbuilding is determined exactly by the demand in state $H$ is extreme. However, a similar outcome

condition,

$$(30) \quad u'(c_{-1}) = \beta \left( \pi^H \left( R_0^H + 1 - \delta^k \right) u'\left( c_0^H \right) + \left( 1 - \pi^H \right) \left( R_0^L + 1 - \delta^k \right) u'\left( c_0^L \right) \right).$$

Online Appendix B.3 completes the characterization, and establishes that there is a demand-driven recession in state $L$ of date $0$ if $\lambda^H$ and $\pi^H$ are sufficiently high.

Next consider a constrained planner who can determine households' date $-1$ allocations, including $h_0, k_0$, but cannot interfere with equilibrium allocations starting date $0$. Like households, the planner also optimally chooses $h_{0,pl} = \left( 1 + \lambda^H \right) h^*$. However, the planner's choice of nonhousing capital, $k_{0,pl}$, is potentially different. Online Appendix B.3 describes the constrained planning problem and characterizes the planner's optimality condition as,

$$(31)$$
$$u'(c_{-1}) = \beta \left( \pi^H \left( R_0^H + 1 - \delta^k \right) u'\left( c_0^H \right) + \left( 1 - \pi^H \right) \left( R_0^L + \left( 1 - \tau_0 \right) \left( 1 - \delta^k \right) \right) u'\left( c_0^L \right) \right).$$

Conditions (30) and (31) are similar except that the planner penalizes the nondepreciated part of the capital in state $L$, since $1 - \tau_0 < 1$, which leads to the following.

PROPOSITION 3 (Restricting Ex-ante Investment):   *Consider the setup with an ex-ante period, characterized in Lemma 3 in Online Appendix B.3. The constrained planner chooses a lower level of investment compared to the competitive equilibrium, $k_{0,pl} < k_0$.*

Intuitively, some of the capital invested at date $-1$ remains nondepreciated at date $0$, which in turn lowers aggregate demand and exacerbates the recession in state $L$. Private agents do not internalize these negative externalities and overinvest in capital from a social point of view. In our stylized model, the inefficiency does not show up in housing capital, because the extreme preferences in (2) imply a corner solution for both the private sector and the planner. In alternative formulations with somewhat elastic housing demand, the planner would optimally restrict ex-ante investment in both types of capital. In fact, Eq. (31) suggests that the externality is particularly strong for more durable types of capital such as housing, because the inefficiency is driven by the nondepreciated part.

Proposition 3 is reminiscent of the results in a recent literature, e.g., Korinek and Simsek (2016) and Farhi and Werning (2016), which investigate the implications of aggregate demand externalities for ex-ante macroprudential policies in financial markets. For instance, Korinek and Simsek (2016) show that, in the run-up to liquidity traps, private agents take on too much debt, because they do not internalize that leverage reduces aggregate demand. We complement this analysis by showing that aggregate demand externalities also create inefficiencies for ex-ante *physical* investment. Our model highlights a distinct mechanism, and generates policy implications that are not the same as the macroprudential policies typically emphasized in this literature. We provide a rationale for restricting ex-ante investment regardless of whether investment is financed by debt or other means.

would also obtain in less extreme versions as long as $\pi^H$ is sufficiently large.

The planner could restrict investment by using a variety of direct policies, e.g., taxes, quantity restrictions, or financing restrictions. A natural question is whether the planner should also use monetary policy. The U.S. Federal Reserve has been criticized for keeping the interest rate low in the run-up to the Great Recession. Our next result investigates whether a contractionary policy that raises the interest rate at date $-1$ above its natural level might be desirable.

PROPOSITION 4 (Jointly Optimal Monetary and Investment Policy): *Consider the setup in Proposition 3. Suppose the planner chooses the interest rate, $r_0$, at date $-1$, in addition to controlling the household's ex-ante allocations. It is optimal for this planner to set $r_0 = r_0^*$ and implement $y_{-1} = S(k_{-1})$.*

Once the investment (restricting) policies are in place, it is optimal for monetary policy to pursue its myopic output stabilization goal described in (10). The constrained efficient outcome (characterized by condition (31)) is to reduce the investment at date $-1$ while increasing consumption, so that there is some reallocation but not a recession at date $-1$. The investment policies implement this outcome by allowing the interest rate to be determined in equilibrium so as to clear the goods market (the zero lower bound constraint does not bind at date $-1$ under our assumptions). In contrast, setting a high level of the interest rate, $r_0 > r_0^*$, reduces investment while also creating an inefficient recession at date $-1$.[26]

Let us summarize the insights from the welfare analysis. Ex-post, once the economy is in the demand-driven slump, welfare can be improved by policies that stimulate investment, including investment in the overbuilt capital. Ex-ante, before the economy enters the slump, welfare can be improved by policies that restrict investment. The optimal ex-post and ex-ante policies share the broad principle that they intertemporally substitute investment from periods that feature efficient outcomes to periods (or states) that feature deficient demand. Intertemporal substitution is less costly for more durable types of capital that deliver a utility flow over a long horizon. Our analysis thus suggests that the policy interventions are particularly desirable for more durable types of capital.

## VI. Conclusion

We have presented a model of investment hangover motivated by the Great Recession that combines Hayekian and Keynesian features. On the Hayekian side, the recession is precipitated by overbuilding of durable capital such as housing, which necessitates a reallocation of resources to other sectors. On the Keynesian side, a constrained monetary policy prevents the interest rate from declining sufficiently, which slows down reallocation and creates an aggregate demand shortage. The demand shortage can also reduce investment in other types of capital (that are not overbuilt) via an accelerator effect, leading to a severe recession. Eventually, other types of investment recover, but the slump in the overbuilt sector continues for a long time.

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[26]That said, if the planner does not have access to the investment policies described above, or faces additional costs in implementing these policies, then she might want to resort to constrained monetary policy as a second-best measure.

The model yields predictions consistent with the broad trends of GDP, housing investment, and nonhousing investment in the Great Recession. In particular, the model explains why both types of investment declined initially, but housing investment recovered much less robustly than nonhousing investment. In a calibration exercise, the overbuilding shock (that we measure from external sources) can match the path of housing prices and investment, but it is not sufficiently strong to trigger the recession by itself. When combined with two other plausible shocks—one to the risk premium and one to household leverage—the overbuilding shock also explains a cumulative 5 percentage point reduction in output. A model with these three shocks can reasonably account for the decline in output and its demand-side components during the Great Recession.

The model also features aggregate demand externalities, with several policy implications for investment. Welfare can be improved by ex-post policies that slow down the decumulation of housing capital, as well as ex-ante policies that restrict the accumulation of capital. These policies intertemporally substitute investment towards periods that feature deficient demand.

Although we have focused on the Great Recession, the model is more widely applicable in environments in which the key assumptions hold: durable capital is overbuilt and monetary policy is constrained. During the times of Hayek and Keynes, speculative overbuilding was seen as a critical impetus to recessions, but the focus was more on railroads and industrial plant than on housing. In the recent European context, overbuilding of houses (or structures) might have contributed to the macroeconomic slump in many countries, and monetary policy was constrained by the currency union as well as the liquidity trap. We leave an elaboration of these applications of our model for future work.

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# Online Appendix for
# 'Investment Hangover and the Great Recession'

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APPENDIX A: CALIBRATION

This appendix describes the details of the calibration exercise in Section IV.

APPROXIMATION OF THE KEYNESIAN MULTIPLIER EMBEDDED IN GHH PREFERENCES. —
In a model with GHH preferences, the labor wedge determines the local output multiplier
with respect to demand shocks. This result was obtained by Auclert and Rognlie (2017),
and in this appendix it is restated in the context of the current paper.

To see why the labor wedge is important, note that holding $r_{t+1}$ (controlled by mon-
etary policy) and $c_{t+1}$ (tomorrow's net consumption) constant, $c_t$ is uniquely pinned
down by the Euler equation in (24). Now, equating production to demand, and writ-
ing $x_t \equiv k_{t+1} - (1 - \delta^k)k_t + i_t^h$ for non-consumption demand, we have

$$y_t - v(l_t) = F(k_t, l_t) - v(l_t) = c_t + x_t$$

Assuming $c_t$ fixed due to the observation above, and holding capital $k_t$ fixed, totally
differentiating gives

$$dl_t = \frac{1}{F_l(k_t, l_t) - v'(l_t)} dx_t$$

and noting that $dy_t = F_l(k_t, l_t)dl_t$, this implies

(A1) $$dy_t = \frac{1}{1 - \frac{v'(l_t)}{F_l(k_t,l_t)}} dx_t \equiv \frac{1}{\tau_t} dx_t$$

where $\tau_t$ is the (conventionally defined) labor wedge at time $t$.

Hence the local response of total output to changes in non-consumption demand (ei-
ther from residential or nonresidential investment), holding inherited capital, monetary
policy, and tomorrow's net consumption fixed, is determined by the inverse of the labor
wedge. If the labor wedge is zero, the implied multiplier is infinite; to obtain a more
realistic multiplier in line with empirical evidence, the labor wedge must be sufficiently
high.

Why is this? If there is no labor wedge ($\tau = 0$), then net output $F(k_t, l_t) - v(l_t)$ is
at a local maximum in $l_t$, and the implied variation in $l_t$ needed to increase net output
is infinite. If there is more of a labor wedge ($\tau > 0$), then net output is still increasing
in $l_t$, implying that less movement in $l_t$ in response to a shock that changes net output is
needed.

CHARACTERIZATION OF THE GENERALIZED MODEL FOR CALIBRATION PURPOSES. — As described in the main text, we generalize the baseline model (with housing adjustment costs) in two ways to make it more suitable for calibration. First, we make a distinction between the risk-free rate and the return to capital. This modification affects the threshold return at which the economy enters the liquidity trap region, as illustrated by Eq. (25), but it otherwise does not change the analysis. Second, we also allow for a labor tax that changes the firm's problem as in (26). With the labor tax, the constrained efficient levels of employment and output are determined by

$$L(k_t) = \arg\max_{\tilde{l}} \left(1 - \tau^l\right) F\left(k_t, \tilde{l}\right) - v\left(\tilde{l}\right),$$

$$\text{and } S(k_t) = F(k_t, L(k_t)) - v(L(k_t)).$$

In particular, the labor tax reduces the labor supply, which in turn lowers output. Note also that, when output is supply determined, the tax parameter corresponds to the labor wedge, $\tau^l = \tau_t$ (cf. Eq. (13)). The remaining equilibrium allocations are characterized by the following system,

$$u'(c_t) = \beta\left(1 + r_{t+1}\right) u'(c_{t+1}),$$

$$i_t^h = i^{h*} + \frac{u'(c_t)}{\psi}\left(Q_t - 1\right) \text{ and } h_{t+1} = h_t\left(1 - \delta^h\right) + i_t^h,$$

$$Q_t \geq \frac{1 - \delta^h}{1 + r_{t+1}} Q_{t+1}, h_{t+1} \geq h^* \text{ and one of the inequalities hold as equality,}$$

$$R(k, L(k)) - \delta^k = r_{t+1},$$

$$y_t = c_t + k_{t+1} - \left(1 - \delta^k\right) k_t + i_t^h,$$

and  $y_t \leq S(k_t), r_{t+1} \geq \phi - \pi$ and one of the inequalities hold as equality.

These conditions are analogous to the equilibrium conditions in Section III, with the difference that the last condition incorporates the more general lower bound on the return to capital in Eq. (25).

CALIBRATING THE OVERBUILDING SHOCK. — We calibrate the magnitude of the over-building shock based on the analysis in Haughwout et al. (2013), who provide two measures of the excess supply of housing units during the Great Recession. Their first measure uses the Census data on housing vacancies. They calculate the stock of vacant housing units in excess of a baseline vacancy rate that we would expect to see in normal market conditions (which the authors estimate based on historical vacancy rates for each housing category). According to this measure, the excess vacant housing units peaked at around 3 million in mid-2010 and remained at around 2 million as of 2012 (see their Figure 2.7).

The second measure of oversupply in Haughwout et al. (2013) compares actual house-hold production with an estimate of housing needs based on historical patterns of house-

hold formation and depreciation. As we describe in the main text, this measure implies around 3.4 million houses were overbuilt by mid-2007. We use this number to calibrate the initial excess supply of housing (see Eq. (27)). Their analysis also suggests that it would take the economy 6 years to work the excess supply (see their Figure 2.8). We use this observation to calibrate the housing adjustment costs in the model so that the adjustment in the model is also completed in 6 years (or 3 periods).

Haughwout et al. (2013) also analyze household formation rates, which speak to the demand for housing in recent years. After adjusting for demographics, they predict that the trend rate of growth of households since the mid-1990s should have been around 1.17 million per year. They then compare the cumulative household formation since 1995 relative to the predicted trend of 1.17 million per year. This analysis illustrates that household formation has been roughly in line with the predicted level until 2007, but it has been consistently below the predicted level in recent years. Using more recent Census data, we find that the household formation averaged 780 thousand per year between the first quarter of 2008 and the third quarter of 2016. This suggests that the low residential investment in recent years is at least in part driven by unusually low demand for housing in the aftermath of the bust (which could be due to, among other things, pessimism about the housing market or credit constraints in the mortgage market). In our calibration exercise, we abstract away from this additional demand shock for housing as it is difficult to quantify.

This appendix completes the analysis of the extensions of the baseline model discussed in the main text. Appendix C contains the proofs of omitted results in the main text as well as this appendix.

### 1. Comparative statics with respect to durability

A distinguishing feature of housing capital is its durability relative to other types of capital. A natural question is whether durability is conducive to triggering a demand-driven recession driven by overbuilding. In this section, we address this question in an extension of the baseline model with two types of housing capital, one more durable than the other. We show that overbuilding the more durable capital (relative to the less durable capital) is more likely to trigger a demand-driven recession.

Consider a slight variant of the model in Section II in which there are two types of housing capital that depreciate at different rates given by $\delta^{h^d}$ and $\delta^{h^n}$, with $\delta^{h^d} < \delta^{h^n}$. Thus, type $d$ (durable) housing capital has a lower depreciation rate than type $n$ (nondurable) housing capital. Suppose the preferences in (2) are modified so that each type has a target level $h^*/2$. Suppose also that $\left(\delta^{h^d} + \delta^{h^n}\right)/2 = \delta^h$ so that the average depreciation rate is the same as before. Let $h_0^d = \left(1 + b_0^d\right)(h^*/2)$ and $h_0^n = \left(1 + b_0^n\right)(h^*/2)$, so that $b_0^d$ and $b_0^n$ capture the overbuilding in respectively durable and nondurable capital. The case with symmetric overbuilding, $b_0^d = b_0^n = b_0$, results in the same equilibrium as in Section II. Our next result investigates the effect of overbuilding one type of capital more than the other.

PROPOSITION 5 (Role of Durability): *Consider the model with two types of housing capital with different depreciation rates. Given the average overbuilding $b_0 = \left(b_0^d + b_0^n\right)/2$, the incidence of a demand-driven recession $\mathbf{1}\left[l_0 < L(k_0)\right]$ is increasing in overbuilding of the more durable housing capital $b_0^d$.*

To obtain an intuition, consider the maximum aggregate demand at date 0, which can be written as [cf. Eq. (19)],

$$(\text{B.1}) \qquad \overline{y}_0 = \overline{k} - \left(1 - \delta^k\right)k_0 + \overline{c}_0 + \delta^h h^* - b_0^d\left(1 - \delta^{h^d}\right)\frac{h^*}{2} - b_0^n\left(1 - \delta^{h^n}\right)\frac{h^*}{2}.$$

Note that $1 - \delta^{h^d} > 1 - \delta^{h^n}$, and thus, overbuilding of the durable housing capital (relative to the nondurable capital) induces a greater reduction in aggregate demand at date 0. Intuitively, depreciation helps to "erase" the overbuilt capital naturally, thereby inducing a smaller reduction in investment and aggregate demand.

### 2. Investment hangover with exogenous monetary policy

A key ingredient of our analysis is constrained monetary policy. In the main text, we focus on the zero lower bound (ZLB) as the source of the constraint. In this section,

we derive the analogue of our main result in Section II in an environment in which the money supply is determined by exogenous forces.

To introduce the money supply, we modify household preferences to introduce the demand for money explicitly. Specifically, the household's optimization problem can now be written as,

$$(\text{B.2}) \qquad \max_{\{l_t, \hat{c}_t, a_{t+1}, M_t\}_t} \sum_{t=0}^{\infty} \beta^t u \left( \hat{c}_t - \upsilon(l_t) + \eta \left( \frac{M_t}{P_t} \right) \right) + u^h \mathbf{1} \left[ h_t \geq h^* \right]$$

$$\text{s.t. } P_t \left( \hat{c}_t + a_{t+1} + i_t^h \right) + M_t = P_t \left( w_t l_t + a_t (1 + r_t) + \Pi_t \right) + M_{t-1},$$

$$\text{and } h_{t+1} = h_t \left( 1 - \delta^h \right) + i_t^h.$$

Here, $P_t$ denotes the aggregate price level. The household money balances are denoted by $M_t$, and the real money balances are given by $M_t/P_t$. The function, $\eta(\cdot)$ is strictly increasing, which captures the transaction services provided by additional real money balances. The household problem is the same as in Section I except for the presence of money balances in preferences as well as the budget constraint. The optimality condition for money balances, $M_t$, implies a money demand equation,

$$(\text{B.3}) \qquad\qquad\qquad \eta' \left( \frac{M_t}{P_t} \right) = \frac{r_{t+1}^n}{1 + r_{t+1}^n}.$$

Here, $1 + r_{t+1}^n = (1 + r_{t+1}) \frac{P_{t+1}}{P_t}$ denotes the nominal interest rate, which captures the opportunity cost of holding money balances (as opposed to interest-bearing assets). The left hand side captures the marginal benefit of holding money balances.[27] The rest of the equilibrium is as described before.

We assume the money supply follows an exogenous path, $\left\{ \overline{M}_t \right\}_{t=0}^{\infty}$. For analytical tractability, we focus on the case in which the money supply is fixed, $\overline{M}_t = \overline{M}$ for each $t$ (the general case is similar). As before, the aggregate price level is also predetermined and constant, $P_t = P$ for each $t$. Combining these assumptions with Eq. $(B.3)$ implies that the nominal interest rate is also constant. There is one degree of freedom because different choices for the aggregate price level (which is a given of this model) lead to different levels for the interest rate. We assume the aggregate price level is such that the interest rate is equal to its steady-state level, that is,[28]

$$(\text{B.4}) \qquad\qquad\qquad r_{t+1} = r_{t+1}^n = 1/\beta - 1 \text{ for each } t.$$

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[27] With our specification, the marginal benefit does not depend the household's consumption or aggregate output. This is slightly different than conventional specifications of money demand but it does not play an important role beyond providing analytical tractability.

[28] This price level can be justified by assuming that the prices were set at a point in the past at which the economy was (and was expected to remain) at a steady state. In view of a New-Keynesian Phillips curve, the firms would not want to change their prices as long as they expected the discounted sum of the output gaps to be equal to zero. When the economy is at a steady state, this requirement implies a zero output gap for each period, which in turn implies the interest rate given by $(B.4)$.

The characterization of the remaining equilibrium allocations then parallels the baseline analysis. We conjecture an equilibrium in which, starting date 1 onwards, the employment and output are at their efficient levels. As before, this implies capital earns its marginal contribution to supply, $R_1 = S'(k_1)$ [cf. (9)]. Combining this with Eq. (6), and using $(B.4)$, we obtain $k_1 = k^*$. That is, the economy reaches the steady-state level of capital in a single period. This determines the investment at date 0 as

$$i_0^k = k_1 - (1 - \delta^k) k_0.$$

Next consider (net) consumption at date 0. Since the economy reaches the steady-state at date 1, we have $c_1 = c^*$. Combining this with the Euler equation and Eq. $(B.4)$, we also obtain $c_0 = c^*$. It follows that aggregate demand and output at date 0 is given by [cf. Eq. (19)],

$$y_0 = k^* - (1 - \delta^k) k_0 + c^* + (\delta^h - b_0 (1 - \delta^h)) h^*.$$

When $y_0 < S(k_0)$, the economy features a demand-driven recession at date 0. This is the case as long as the amount of overbuilding $b_0$ exceeds a threshold level [cf. (20)],

$$\bar{b}_0 \equiv \frac{k^* - (1 - \delta^k) k_0 + c^* + \delta^h h^* - S(k_0)}{(1 - \delta^h) h^*}.$$

It can also be checked that, it the initial capital stock is at its steady-state level $k_0 = k^*$, then the threshold is zero, $\bar{b}_0 = 0$: that is, any amount of overbuilding triggers a recession.

Hence, our main result generalizes to a setting with exogenous (and fixed) money supply. Intuitively, the key to the argument is that monetary policy is constrained and cannot lower the interest rate sufficiently to counter the aggregate demand reduction due to overbuilding. When monetary policy is exogenous—as in the case of an exogenous money supply, it is naturally constrained and cannot lower the interest rate in response to shocks. In fact, overbuilding in this case leads to a deeper recession because the nominal interest rate remains above zero during the recession, whereas monetary policy in the main text partially fights the recession by lowering the nominal interest rate to zero.

### 3. Policy analysis with separable preferences

We next complete the analysis of the model with separable preferences described and used in Section V. We first establish the analog of Proposition 1 for this setting. To this end, let $\bar{c}_0$ and $\bar{k}$ respectively denote the maximum level of consumption and investment characterized in Section II. The aggregate demand is then bounded from above, $y_0 \leq \bar{y}_0$, where

(B.5) $$\bar{y}_0 \equiv \bar{k} - (1 - \delta^k) k_0 + \bar{c}_0 + (\delta^h - b_0 (1 - \delta^h)) h^*.$$

as in Eq. (19) in the main text.

Next consider the efficient level of employment at date 0. The efficiency implies the household's intratemporal condition holds, $w_0 u'(c_0) = v'(l_0)$, and the equilibrium wage level is determined by the labor's marginal product, $w_0 = F_l(k_0, l_0)$. Combining these conditions is equivalent to setting the labor wedge to zero, where the labor wedge is now given by,

$$(B.6) \qquad \tau_0 = 1 - \frac{v'_0(l_0)}{u'(c_0) F_l(k_0, l_0)}.$$

Let $L_0(k_0)$ denote the efficient level of output at date 0 (when there is a liquidity trap) characterized by setting $\tau_0 = 0$ when $c_0 = \bar{c}_0$. This also implies an efficient level of output denoted by, $S_0(k_0) = F(k_0, L_0(k_0))$.

As in Section II, the equilibrium depends on a comparison of the maximum level of demand, $\bar{y}_0$, with the efficient supply, $S_0(k_0)$. Let $\bar{b}_0^{sep}$ denote the threshold level of overbuilding that ensures $\bar{y}_0 = S_0(k_0)$, that is,

$$(B.7) \qquad \bar{b}_0^{sep} = \frac{\bar{k} - \left(1 - \delta^k\right) k_0 + \bar{c}_0 + \delta^h h^* - S_0(k_0)}{\left(1 - \delta^h\right) h^*}.$$

We then have the following analogue of Proposition 1.

LEMMA 2: *Consider the modified model with separable preferences at date 0. The competitive equilibrium decumulates the excess housing capital in a single period, $h_1 = h^*$. If the overbuilding is sufficiently large, $b_0 > \bar{b}_0^{sep}(k_0)$, then the date 0 equilibrium features a demand-driven recession with,*

$$r_1 = 0, \quad \tau_0 > 0, \quad y_0 = \bar{y}_0 < S_0(k_0), \text{ and } l_0 < L_0(k_0).$$

EX-POST WELFARE ANALYSIS. — Next suppose the overbuilding is sufficiently large so that the economy is in a recession. We next respectively define the household's and the planner's value functions and derive their optimality conditions. Note that choosing $h_1 < h^*$ is sub-optimal in view of the preferences (2). We thus consider the value functions over the region $h_1 \geq h^*$.

The household's problem can then be written as (cf. problem (5)),

$$\begin{aligned} W_0(h_1) &= \max_{\{c_t, a_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t.} \quad & c_t + a_{t+1} + h_{t+1} = e_t + a_t(1 + r_t) + \Pi_t + \left(1 - \delta^h\right) h_t \\ \text{given } h_0 &\geq h^*, h_1 \geq h^* \text{ and } h_t = h^* \text{ for each } t \geq 2. \end{aligned}$$

Using the envelope theorem, we obtain,

$$\frac{dW_0(h_1)}{dh_1}\Big|_{h_1=h^*} = \beta u'(c_1)\left(1-\delta^h\right) - u'(c_0).$$

Combining this with the Euler equation, $u'(c_0) = \beta(1+r_1)u'(c_1)$, establishes Eq. (28).

Next consider a constrained planner who can (only) control housing investment at date 0. When $h_1$ is in a neighborhood of $h^*$, the constrained planning problem can be written as,

$$(B.8) \qquad W_{0,pl}(h_1) = \max_{c_0,k_1,y_0,l_0} u(c_0) - v_0(l_0) + \beta V(k_1,h_1),$$

$$\text{s.t. } k_1 = \bar{k} \text{ and } u'(c_0) = \beta u'(C(h_1)),$$

$$(B.9) \qquad \text{and } y_0 = F(k_0,l_0) = k_1 - \left(1-\delta^k\right)k_0 + c_0 + h_1 - \left(1-\delta^h\right)(1+b_0)h^*.$$

Here, $V(k_1,h_1)$ denotes the efficient value function characterized as the solution to problem $(C.1)$, and $C(h_1)$ denotes the efficient level of consumption. The second line captures the zero lower bound constraint, which implies that nonhousing investment and consumption are determined by the zero interest rate. The third line captures that output and employment are determined by the aggregate demand at date 0. Importantly, output is increasing in $h_1$ because a greater level of housing investment increases aggregate demand.

To derive the optimality condition for problem $(B.8)$, note that the capital stocks $k_0$ and $k_1 = \bar{k}$ are constant, and that the remaining variables, $c_0(h_1)$, $y_0(h_1)$, $l_0(h_1)$, are determined as implicit functions of $h_1$. Implicitly differentiating the aggregate demand constraint $(B.9)$ with respect to $h_1$, we obtain,

$$\frac{dl_0}{dh_1} = \frac{1+\frac{dc_0}{dh_1}}{F_l(k_0,l_0)} = \left(1+\frac{dc_0}{dh_1}\right)\frac{(1-\tau_0)u'(c_0)}{v'(l_0)}.$$

Here, the second equality substitutes the labor wedge from Eq. $(B.6)$. Using problem $(C.1)$ along with the envelope theorem, we also obtain,

$$\frac{dV_1(k_1,h_1)}{dh_1} = \left(1-\delta^h\right)u'(c_1) = \left(1-\delta^h\right)\frac{u'(c_0)}{\beta}.$$

Here, the second equality uses the Euler equation. Differentiating the objective function of problem $(B.8)$ with respect to $h_1$, and using these expressions, we obtain,

$$\begin{aligned} \frac{dW_{0,pl}(h_1)}{dh_1} &= u'(c_0)\frac{dc_0}{dh_1} - v_0'(l_0)\frac{dl_0}{dh_1} + \beta\frac{dV_1(k_1,h_1)}{dh_1}, \\ &= u'(c_0)\left(\frac{dc_0}{dh_1} - \left(1+\frac{dc_0}{dh_1}\right)(1-\tau_0) + 1 - \delta^h\right). \end{aligned}$$

Rearranging terms establishes Eq. (29). Using this expression, Appendix C proves Proposition 2 and completes the welfare analysis in Section V.A.

EX-ANTE WELFARE ANALYSIS. — Next consider the ex-ante welfare analysis in Section V.B. Recall that the representative household optimally chooses $h_0 = h^* \left(1 + \lambda^H\right)$, along with $k_0$ characterized as the solution to (30). The representative household recognizes that the rental rate of capital in state $L$, $R_0^L$, is below its efficient level (due to the demand shortage). This might induce her to choose a lower level of $k_0$ as a precaution. A sufficiently low level of $k_0$ can, in turn, raise the aggregate demand and prevent the demand-driven recession [cf. Eq. ($B$.7)]. Nonetheless, the following result establishes that the economy experiences a recession in state $L$, as long as the probability of the state is sufficiently low, and the demand for housing in the counterfactual state $H$ is sufficiently high.

LEMMA 3: *Consider the modified model with the ex-ante date* $-1$*, with the initial conditions,* $h_{-1} = h^* \left(1 + \lambda^H\right)$ *and* $k_{-1} = k^*$*. Suppose* $\lambda^H > \overline{b}_0^{sep} \left(k^*\right)$*, where* $\overline{b}_0^{sep} \left(k^*\right)$ *denotes the overbuilding threshold in* ($B$.7) *given* $k_0 = k^*$*. There exists* $\overline{\pi} < 1$ *such that, if* $\pi^H \in \left(\overline{\pi}, 1\right)$*, then the equilibrium features a demand-driven recession in state* $L$ *of date* 0 *(but not in any other dates or states).*

The equilibrium path starting with the high-demand state $H$ of date 0 is straightforward. It solves the neoclassical planning problem ($C$.1) with a steady level of housing investment given by, $i_t^h = \delta \left(1 + \lambda^H\right) h^*$ for each $t \geq 0$. The zero lower bound does not bind and the rental rate of capital is given by $R_0^H = S'(k_0)$. The equilibrium path starting with the low-demand state $L$ of date 0 is characterized as in Lemma 2 given the (endogenous) level of overbuilding, $b_0 = \lambda^H$.

Next consider a constrained planner who can (only) control households' date $-1$ allocations. As described in the main text, the planner optimally chooses $h_{0,pl} = h_0 = \left(1 + \lambda^H\right) h^*$. However, the planner's choice of nonhousing capital, $k_{0,pl}$, is potentially different. To characterize this choice, let $V_0^H(k_0, h_0)$ and $V_0^L(k_0, h_0)$ denote the welfare of the representative household in respectively states $H$ and $L$ of date 0. The ex-ante constrained planning problem can then be written as,

(B.10)  $$\max_{c_{-1}, k_0} u\left(c_{-1}\right) + \beta \left(\pi^H V_0^H\left(k_0, h_0\right) + \left(1 - \pi^H\right) V_0^L\left(k_0, h_0\right)\right),$$

$$\text{s.t.} \quad c_{-1} + k_0 + h_{0,pl} = S\left(k_{-1}\right) + \left(1 - \delta^k\right) k_{-1} + \left(1 - \delta^h\right) h_{-1}.$$

In particular, the planner optimally trades off the ex-ante consumption, $c_{-1}$, with investment, $k_0$, evaluating the benefits of the latter in the competitive equilibrium that will obtain in each state. The optimality condition for the problem is then given by

(B.11)  $$u'\left(c_{-1}\right) = \beta \left(\pi^H \frac{dV_0^H\left(k_0, h_0\right)}{dk_0} + \left(1 - \pi^H\right) \frac{dV_0^L\left(k_0, h_0\right)}{dk_0}\right).$$

We next derive $\frac{dV_0^H(k_0,h_0)}{dk_0}$ and $\frac{dV_0^L(k_0,h_0)}{dk_0}$, and establish Eq. (31). If state $H$ is realized, then the equilibrium solves the analog of problem $(C.1)$ (with appropriate modifications to capture the higher target level, $(1 + \lambda^H) h^*$). Then, the envelope theorem implies,

$$\frac{dV_0^H(k_0, h_0)}{dk_0} = (S'(k_0) + 1 - \delta^k) u'(c_0^H).$$

Suppose instead state $L$ is realized. The continuation allocation is characterized by Lemma 2, and it solves problem $(B.8)$ with $h_1 = h^*$ (since we rule out ex-post policies). This problem implies that the following variables are constant, $k_1 = \bar{k}$, $c_0 = \bar{c}_0$, $h_1 = h^*$ (and thus, the continuation value $V_1$ is also constant). In contrast, output and employment, $y_0(k_0)$, $l_0(k_0)$, are determined as implicit functions of $k_0$. Implicitly differentiating the aggregate demand constraint $(B.9)$ with respect to $k_0$, we obtain,

$$\frac{dl_0}{dk_0} = -\frac{F_k(k_0, l_0) + (1 - \delta^k)}{F_l(k_0, l_0)} = -\left(F_k(k_0, l_0) + (1 - \delta^k)\right) \frac{(1 - \tau_0) u'(\bar{c}_0)}{v'(l_0)}.$$

Here, the second equality substitutes the labor wedge from Eq. $(B.6)$. Differentiating the objective function with respect to $k_0$, and using this expression, we further obtain,

$$\frac{dV_0^L(k_0, h_0)}{dk_0} = -v_0'(l_0) \frac{dl_0}{dk_0} = (1 - \tau_0)\left(F_k(k_0, l_0) + (1 - \delta^k)\right) u'(\bar{c}_0).$$

Plugging in $R_0^L = (1 - \tau_0) F_k(k_0, l_0)$ from Lemma 2 implies,

$$\frac{dV_0^L(k_0, h_0)}{dk_0} = \left(R_0^k + (1 - \tau_0)(1 - \delta^k)\right) u'(\bar{c}_0).$$

Plugging the expressions for $\frac{dV_0^H(k_0,h_0)}{dk_0}$ and $\frac{dV_0^L(k_0,h_0)}{dk_0}$ into $(B.11)$ implies the planner's optimality condition (31). Appendix C proves Propositions 3 and 4, and completes the welfare analysis in Section V.B.

This appendix presents the omitted characterizations and proofs.

### 1. *Proofs for the baseline model analyzed in Sections I and II*

**Characterization of the efficient benchmark.** Consider a planner that maximizes households' welfare starting date $t$ onwards, given the initial state $h_t, k_t$, and the feasibility constraints of the economy. The planner's problem can then be written as,

$$\max_{\{\hat{c}_{\tilde{t}}, l_{\tilde{t}}, k_{\tilde{t}+1}, \tilde{h}_{t+1}, [l_{\tilde{t}}(v), k_{\tilde{t}}(v)]_v\}_{\tilde{t}=t}^{\infty}} \sum_{\tilde{t}=t}^{\infty} \beta^{\tilde{t}} \left( u\left(\hat{c}_{\tilde{t}} - v\left(l_{\tilde{t}}\right)\right) + u^h \mathbf{1}\left[h_t \geq h^*\right]\right),$$

$$\text{s.t. } \hat{c}_{\tilde{t}} + k_{\tilde{t}+1} + h_{\tilde{t}+1} \leq \hat{y}_{\tilde{t}} + \left(1 - \delta^k\right) k_{\tilde{t}} + \left(1 - \delta^h\right) h_{\tilde{t}}, \text{ where}$$

$$\hat{y}_{\tilde{t}} = \left(\int_0^1 \left(F\left(k_{\tilde{t}}\left(v\right), l_{\tilde{t}}\left(v\right)\right)\right)^{\frac{\varepsilon-1}{\varepsilon}} dv\right)^{\varepsilon/(\varepsilon-1)}, k_{\tilde{t}} = \int k_{\tilde{t}}\left(v\right) dv, \text{ and } l_{\tilde{t}} = \int l_{\tilde{t}}\left(v\right) dv.$$

By concavity, the planner chooses $k_{\tilde{t}}\left(v\right) = k_{\tilde{t}}$ and $l_{\tilde{t}}\left(v\right) = l_{\tilde{t}}$ for each $\tilde{t}$. The optimality condition for labor then implies Eq. (9). Combining these observations, the planner's problem reduces to the neoclassical planning problem,

$$(C.1) \quad V\left(k_t, h_t\right) = \max_{\{c_{\tilde{t}}, k_{\tilde{t}+1}, h_{t+1}\}_{\tilde{t}=t}^{\infty}} \sum_{\tilde{t}=t}^{\infty} \beta^{\tilde{t}} \left(u\left(c_{\tilde{t}}\right) + u^h \mathbf{1}\left[h_t \geq h^*\right]\right),$$

$$\text{s.t.} \quad c_{\tilde{t}} + k_{\tilde{t}+1} - \left(1 - \delta^k\right) k_{\tilde{t}} + h_{\tilde{t}+1} - \left(1 - \delta^h\right) h_{\tilde{t}} = S\left(k_{\tilde{t}}\right).$$

Here, the function $S\left(\cdot\right)$ describes the supply-determined net output defined in (9).

**Equilibrium in the aftermath of overbuilding.** Suppose the economy reaches date 1 with $h_1 = h^*$ and $k_1 \leq \bar{k}$. We claim that the continuation equilibrium is the same the efficient benchmark. To this end, consider the solution to the planner's problem $(C.1)$ starting with $h_1 = h^*$ and $k_1 \leq \bar{k}$. We conjecture a solution in which $h_{t+1} = h^*$ for each $t \geq 1$, as in (3), and the remaining allocations are characterized as the solution to the neoclassical system,

$$(C.2) \qquad\qquad S\left(k_{\tilde{t}}\right) = c_{\tilde{t}} + k_{\tilde{t}+1} - \left(1 - \delta^k\right) k_{\tilde{t}} + \delta^h h^*,$$
$$u'\left(c_{\tilde{t}}\right) = \beta\left(1 + S'\left(k_{\tilde{t}}\right) - \delta^k\right) u'\left(c_{\tilde{t}+1}\right),$$

together with a standard transversality condition. The steady-state to this system is characterized by,

$$\beta\left(1 - \delta^k + S'\left(k^*\right)\right) = 1 \text{ and } S\left(k^*\right) = c^* + \delta^k k^* + \delta^h h^*.$$

We assume the parameters satisfy, $\min\left(S\left(k_0\right), S\left(k^*\right)\right) > \delta^k k^* + \delta h^*$, which ensures

that the economy can afford the required investment at all periods. Then, using standard arguments, there is a unique interior path that solves the system in $(C.2)$ and converges to the steady state. Moreover, since capital converges monotonically to its steady-state level, and since we have $k_1 \leq \bar{k}$ and $k^* < \bar{k}$, we also have $k_{t+1} \leq \bar{k}$ for each $t \geq 1$. This in turn implies the interest rate satisfies, $r_{t+1} = S'(k_{t+1}) - 1 \geq S'(\bar{k}) - 1 = 0$ for each $t \geq 1$.

In particular, the implied real interest rate is nonnegative along the socially optimal path, which has two implications. First, the planner finds it optimal to choose $h_{t+1} = h^*$ as we have conjectured (since the gross return on investment, $1 + r_{t+1}$, exceeds the return on empty houses, $1 - \delta^h$). Second, and more importantly, the lower bound constraint (7) does not bind along the socially optimal path. This implies that the monetary policy rule in (10) replicates the dynamically efficient outcomes. That is, the competitive equilibrium from date 1 onwards (starting $h_1 = h^*$ and $k_1 \leq \bar{k}$) coincides with the efficient benchmark. This completes the characterization of the equilibrium in the aftermath of overbuilding. ∎

**Proof of Lemma 1.** First consider the case $r_{t+1} > 0$. In this case, monetary policy implements the efficient allocation with $l_t = L(k_t)$ and $y_t = S(k_t)$. In addition, the first order conditions for problems (9) and (4) further imply, $F_l(k_t, L(k_t)) = v'(L(k_t)) = w_t$. Combining this with Eq. (12) implies that the labor wedge is zero, $\tau_t = 0$. Combining Eqs. (12) and (9) then imply the rental rate of capital is given by $F_k(k_t, L(k_t)) = S'(k_t)$, completing the proof for the first part.

Next consider the case $r_{t+1} = 0$. In this case, Eq. (12) implies $F_l(k_t, l_t) \geq v'(l_t)$. This in turn implies that $l_t \in [0, L(k_t)]$. By feasibility, net output satisfies

$$y_t = c_t + i_t^h + i_t^h = F(k_t, l_t) - v(l_t).$$

This right hand side is strictly increasing in $l_t$ over the range $[0, L(k_t)]$. The minimum and the maximum are respectively given by 0 and $S(k_t)$, which implies $y_t \in [0, S(k_t)]$. Moreover, given $y_t$ that satisfies these resource constraints, there is a unique $l_t$ that solves (11). Combining this with Eq. (12), we further obtain the labor wedge as, $1 - \tau_t = \frac{v'(l_t)}{F_l(k_t, l_t)}$. Plugging this into Eq. (12) for capital, we obtain the rental rate of capital as, $R(k_t, l_t) = \frac{v'(l_t)}{F_l(k_t, l_t)} F_k(k_t, l_t)$. It can be checked that $R_k < 0$, $R_l > 0$ over $l \in [0, L(k_t)]$, and that $R(k_t, L(k_t)) = S'(k_t)$, completing the proof. ∎

**Proof of Proposition 1.** As we have shown above, the equilibrium at date 1 starting with $h_1 = h^*$ and $k_1 \leq \bar{k}$ coincides with the efficient benchmark. Note also that, by standard arguments, the neoclassical system in $(C.2)$ can be described by an increasing consumption function, $c_1 = C(k_1)$.

To characterize the equilibrium at date 0, we define $K_1(r_0)$ for each $r_0 \geq 0$ as the solution to

$$S'(K_1(r_0)) - \delta^k = r_0.$$

Note that $K_1(r_0)$ is decreasing in the interest rate, with $K_1(0) = \bar{k}$ and

$\lim_{r_0 \to \infty} K_1(r_0) = 0$. Similarly, define the function $C_0(r_0)$ as the solution to the Euler equation

$$u'(C_0(r_0)) = \beta(1 + r_0) u'(C(K_1(r_0))).$$

Note that $C_0(r_0)$ is decreasing in the interest rate, with $C_0(0) = \bar{c}_0$ and $\lim_{r_0 \to \infty} C_0(r_0) = 0$. Finally, define the aggregate demand function

$$Y_0(r_0) = C_0(r_0) + K_1(r_0) - \left(1 - \delta^k\right) k_0 + i_0^h.$$

Note that $Y_0(r_0)$ is also decreasing in the interest rate, with

$$Y_0(0) = \bar{y}_0 \text{ and } \lim_{r_0 \to \infty} Y_0(r_0) = i_0^h - \left(1 - \delta^k\right) k_0.$$

Next consider the date 0 equilibrium for the case $b_0 \leq \bar{b}_0$. Note that this implies $S(k_0) \leq \bar{y}_0 = Y_0(0)$, and that we also have $\lim_{r_0 \to \infty} Y_0(r_0) < S(k_0)$ (since we assume housing investment is feasible). By the intermediate value theorem, there is a unique equilibrium interest rate $r_0 \in [0, \infty)$ such that $Y_0(r_0) = S(k_0)$. The equilibrium features $c_0 = C_0(r_0)$ and $K_1(r_0) = k_1$, along with $y_0 = S(k_0)$ and $l_0 = L(k_0)$.

Next consider the date 0 equilibrium for the case $b_0 > \bar{b}_0$. In this case, $Y_0(0) < S(k_0)$. Thus, the unique equilibrium features $r_0 = 0$ and $y_0 = \bar{y}_0 < S(k_0)$. Consumption and investment are given by $c_0 = \bar{c}_0$ and $k_1 = \bar{k}_1$. Labor supply $l_0$ is determined as the unique solution to (11) over the range $l_0 \in (0, L(k_0))$. Finally, Eq. $(B.5)$ implies the equilibrium output, $y_0 = \bar{y}_0$, is declining in the initial overbuilding $b_0$.

In either case, it can also be checked that the economy reaches date 1 with $h_1 = h^*$ and $k_1 \geq \min(k_0, k^*)$. Thus, the continuation equilibrium is characterized as described above, completing the proof. ∎

**Proof of Proposition 5.** Note that the recession is triggered if $\bar{y}_0 < S(k_0)$, where $\bar{y}_0$ is given by Eq. $(B.1)$. Since $1 - \delta^{h^d} > 1 - \delta^{h^n}$, increasing $b_0^d$ (while keeping $b_0 = \left(b_0^d + b_0^n\right)/2$ constant) reduces $\bar{y}_0$, proving the result. ∎

### 2. Proofs for the policy analysis in Section V and Appendix B.3

**Proof of Lemma 2.** Most of the proof is described in Appendix B.3. If $b_0 < \bar{b}_0^{sep}$, then the maximum aggregate demand is above the efficient level, $\bar{y}_0 > S_0(k_0)$. In this case, the zero lower bound constraint does not bind and outcomes are efficient. If instead $b_0 > \bar{b}_0^{sep}$, then output is below the efficient level and it is determined by aggregate demand, $y_0 = \bar{y}_0 < S_0(k_0)$. The employment is also below the efficient level, $l_0 < L_0(k_0)$, and it is characterized by solving, $y_0 = \bar{y}_0 = F(k_0, l_0)$. The labor wedge is characterized by solving, $1 - \tau_0 = \frac{v_0'(l_0)}{F_l(k_0, l_0) u'(\bar{c}_0)}$, and it satisfies $\tau_0 > 0$.

**Proof of Proposition 2.** We first show that the planner's marginal utility, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}\big|_{h_1 = h^*}$, is increasing in the labor wedge, $\tau_0$. Note that the Euler equation in problem $(B.8)$

implies,

$$\frac{dc_0}{dh_1}|_{h_1=h^*} = \frac{\beta u'' (C(h^*))}{u'' (\overline{c}_0)} C'(h^*) > 0.$$

Here, the inequality follows because the solution to the neoclassical problem $(C.1)$ implies $C'(h^*) > 0$. Note also that the derivative $\frac{dc_0}{dh_1}|_{h_1=h^*}$ is independent of $b_0$ or $\tau_0$. Combining this with Eq. (29) proves that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$ is increasing $\tau_0$.

Next note from the proof of **2** that the labor wedge, $\tau_0$, is strictly decreasing in aggregate demand, $y_0 = \overline{y}_0$. Since the maximum demand, $\overline{y}_0$, in Eq. $(B.5)$ is strictly decreasing in overbuilding, $b_0$, this implies that the labor wedge is strictly increasing in overbuilding, $b_0$. This in turn implies that the planner's marginal utility, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$, is strictly increasing in $b_0$. It can also be checked that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} > 0$ for sufficiently high levels of $b_0$. Let $\tilde{b}_0 > \overline{b}_0^{sep}$ denote the level of overbuilding such that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} = 0$. It follows that, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} > 0$ if and only if $b_0 > \tilde{b}_0$. This also implies $h_{1,pl} > h^*$ if and only if $b_0 > \tilde{b}_0$. ∎

**Proof of Lemma 3.** First consider the limiting case with $\pi^H = 1$. In this case, given the initial conditions, the economy is at an efficient steady-state with,

$$h_t = h^* (1 + \lambda^H), k_t = k^* \text{ and } c^* = S(k^*) - \delta^h (1 + \lambda^H) h^* - \delta^k k^*.$$

In particular, the competitive equilibrium features $k_0 = k^*$. In this equilibrium, the economy does not feature a demand shortage at date 0 or state $H$ of date 1. In fact, we have $r_1 = r_2^H = 1/\beta > 0$. However, since $\lambda^H > \overline{b}_0^{sep}(k^*)$, the economy features a demand shortage in the (zero probability) state $L$.

Next note that the capital choice in competitive equilibrium is a continuous function of the probability of the high state, $k_0(\pi^H)$. By Eq. $(B.7)$, $\overline{b}_0^{sep}(k_0)$ is also a continuous function of $k_0$. It follows that there exists $\overline{\pi}^1$ (which could also be $\overline{\pi}^1 = 0$) such that $\lambda^H > \overline{b}_0^{sep}(k^*)$ if and only if $\pi^H > \overline{\pi}^1$. Similarly, note that the interest rates $r_1$ and $r_2^H$ are also continuous functions of $\pi^H$. Using continuity once again, there exists $\overline{\pi}^2 < 1$ (which could also be $\overline{\pi}^2 = 0$) such that the economy does not feature a demand shortage at date 0 or at state $H$ if and only if $\pi^H > \overline{\pi}^2$. Taking $\overline{\pi} = \max(\overline{\pi}^1, \overline{\pi}^2)$ proves the statement. ∎

**Proof of Proposition 3.** The planner's optimality condition (31) implies $k_{0,pl} < k_0$ since $\tau_0 > 0, \pi^H > 0$, and $1 - \delta^k > 0$. ∎

**Proof of Proposition 4.** In this case, the difference is that the planner can also control the ex-ante employment and net output, $l_{-1}, y_{-1}$, by deviating from the monetary policy in (10). Thus, the analogue of the planner's problem in $(B.10)$ is given by,

$$\max_{l_{-1}, y_{-1}, c_{-1}, k_0} u\left(c_{-1}\right) + \beta \left(\pi^H V_0^H\left(k_0, h_0\right) + \left(1 - \pi^H\right) V_0^L\left(k_0, h_0\right)\right),$$

$$\text{s.t.} \quad c_{-1} + k_0 + h_{0,pl} = S\left(k_{-1}\right) + \left(1 - \delta^k\right) k_{-1} + \left(1 - \delta^h\right) h_{-1},$$

$$\text{and } y_{-1} = F\left(k_{-1}, l_{-1}\right) - v\left(l_{-1}\right) \le S\left(k_{-1}\right).$$

It is easy to check that the first order conditions maximize the net output, $y_{-1} = S\left(k_{-1}\right)$ and $l_{-1} = L\left(k_{-1}\right)$. This in turn leads to the same problem $(B.10)$ as before, as well as the same first order conditions $(31)$. In particular, the planner sets the interest rate, $r_0 = r_0^*$, which (by definition) replicates the statically efficient allocations at date $-1$, completing the proof. ∎